## Assignment 4: Probability Distributions

## Expectations

| Assessment | KU: | APP: | T/PS: |
| :--- | :--- | :--- | :--- |

Through this assignment, you will have the opportunity to...
$\qquad$ demonstrate an understanding of discrete probability distributions (specifically, binomial and geometric)
$\qquad$ represent discrete probability distributions numerically,graphically, and algebraically
$\qquad$ determine expected values
$\qquad$ solve related problems from a variety of applications (involving binomial and geometric distributions)

## Instructions

## Part A-Knowledge \& Understanding (KU)

Answer each of the problems that follow. If using Onenote Classroom Notebook, answer between each problem; if completing by hand, do so on lined paper and be prepared to hand in your assignment sheet with your completed assignment. Check the success criteria, below this problem set, for hints as to what you should try to emphasize in your solutions.

1. A random variable $X$ is defined as the number of heads observed when a coin is tossed 4 times. The probability distribution for this random variable is shown below.

| $X$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $P(X)$ | $\frac{1}{16}$ | $\frac{4}{16}$ | $\frac{6}{16}$ | $\frac{4}{16}$ | $\frac{1}{16}$ |

Determine the expected value for this probability distribution.

2i) Which expression describes the probability of $k$ " 3 s " being rolled on 20 successive rolls of a six-sided die? Justify your response.
a. $\binom{20}{k}\left(\frac{1}{6}\right)^{k}\left(\frac{5}{6}\right)^{20-k}$
b. $\binom{20}{k}\left(\frac{5}{6}\right)^{k}\left(\frac{1}{6}\right)^{20-k}$
c. $\binom{20}{k}\left(\frac{3}{6}\right)^{k}\left(\frac{3}{6}\right)^{20-k}$
d. $\binom{20}{3}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{17}$
ii) Why is the scenario in part i) able to be modeled by a binomial distribution? Explain briefly.
3. Consider this question: What is the expected number of failures in 100 launches of a rocket that has a failure rate of $1.5 \%$ ? Explain why this problem does not fit a geometric distribution and how it could be rewritten so that it does.

## Part A-Success Criteria

Before submitting your completed assignment, ensure that you have considered the following:
Did I ...

|  | Category | Criteria |  |  |  |
| :---: | :---: | :--- | :--- | :--- | :--- |
| \#1 | KU | Use the formula, $P\left(X=x_{i}\right)=\sum_{\mathrm{i}=1}^{\mathrm{n}} x_{i} \cdot p\left(x_{i}\right)$, for the <br> expected value of a probability distribution? | Approaching | On <br> Onget | Working <br> to Exceed |
| \#2 | KU | i) Consider what each aspect of the formula represents and <br> use them accordingly to explain? <br> ii) Discuss the criteria that specify a binomial distribution? | Approaching | On <br> Target | Working <br> to Exceed |
| \#3 | KU | Consider what is known about the number of trials in a <br> geometric distribution? | Approaching | On <br> Target |  |

## Part B-Application (APP)

Answer each of the problems that follow. If using Onenote Classroom Notebook, answer between each problem; if completing by hand, do so on lined paper and be prepared to hand in your assignment sheet with your completed assignment. Check the success criteria, below this problem set, for hints as to what you should try to emphasize in your solutions.

## Problems

1. A game is played by drawing cards from a deck that has all the face cards removed including the aces. The player draws a card and is paid the face value of the card in dollars. What is the expected value of this game?
2. The probability of recovering after a particular type of experimental surgery is 0.4 . If 3 patients undergo this operation, ...
a) Complete the binomial probability distribution table for the number of recovering patients. In this case, $S$ denotes recovery.

| \# of successes | Probability Expression | Value |
| :---: | :---: | :---: |
| FFF |  |  |
| SFF | $\binom{3}{2} 0.4^{2} 0.6$ | 0.432 |
|  |  |  |
|  |  |  |

b) Determine the expected number of recoveries for this type of experimental surgery.
3. If the distribution in $\# 2$ were geometric (i.e., with an unspecified number of people), what would be the expected number of surgeries required before experiencing success (i.e., before experiencing a recovery)?
4. Suppose that 1 out of 50 cards in a scratch-and-win promotion gives a prize.
a) What is the probability of winning on your fourth try?
b) What is the expected number of cards you would have to try before winning?

## Part B-Success Criteria

Before submitting your completed assignment, ensure that you have considered the following:
Did I ...

|  | Category | Criteria |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| \#1 | APP | Use (correctly) the formula, $P\left(X=x_{i}\right)=\sum_{\mathrm{i}=1}^{\mathrm{n}} x_{i} \cdot p\left(x_{i}\right)$, for <br> the expected value of a probability distribution? | Approaching | On <br> Target | Working <br> to Exceed |
| \#2 | APP | Consider how I might use patterning to complete the <br> column "Probability Expression"? And correctly evaluate <br> each expression? | Approaching | On <br> Target |  |
| \#3 | APP | Correctly use the formula for calculating the expected value <br> of a geometric distribution? | Approaching | On <br> Target |  |
| \#4 | APP | -Consider the nature of this distribution-binomial or <br> geometric (and justify my choice of formula(s))? <br> -Use appropriate formula correctly? | Approaching | On <br> Target | Working <br> to Exceed |

