Name: Solutions	Date:

Assignment 4: Probability Distributions

Expectations

Assessment	KU:	APP:	T/PS:

Through this assignment, you will have the opportunity to…

_____ demonstrate an understanding of discrete probability distributions (specifically, binomial and geometric)

_____ represent discrete probability distributions numerically~~, graphically,~~ and algebraically

_____ determine expected values

_____ solve related problems from a variety of applications (involving binomial and geometric distributions)

Instructions

Part A-Knowledge & Understanding (KU)
Answer each of the problems that follow. If using Onenote Classroom Notebook, answer between each problem; if completing by hand, do so on lined paper and be prepared to hand in your assignment sheet with your completed assignment. Check the success criteria, below this problem set, for hints as to what you should try to emphasize in your solutions.

1. A random variable X is defined as the number of heads observed when a coin is tossed 4 times. The probability distribution for this random variable is shown below.

$X=x_i$	$0=x_1$	$1=x_2$	$2=x_3$	$3=x_4$	$4=x_5$
$P(X)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Determine the expected value for this probability distribution.

2i) Which expression describes the probability of k "3s" being rolled on 20 successive rolls of a six-sided die? Justify your response.

a. $\binom{20}{k}\left(\frac{1}{6}\right)^k\left(\frac{5}{6}\right)^{20-k}$

b. $\binom{20}{k}\left(\frac{5}{6}\right)^k\left(\frac{1}{6}\right)^{20-k}$

c. $\binom{20}{k}\left(\frac{3}{6}\right)^k\left(\frac{3}{6}\right)^{20-k}$

d. $\binom{20}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{17}$

ii) Why is the scenario in part i) able to be modeled by a binomial distribution? Explain briefly.

3. Consider this question: What is the expected number of failures in 100 launches of a rocket that has a failure rate of 1.5%? Explain why this problem does not fit a geometric distribution and how it could be re-written so that it does.

Part A-Success Criteria

Before submitting your completed assignment, ensure that you have considered the following:

Did I ...

	Category	Criteria			
#1	KU	Use the formula, $P(X = x_i) = \sum_{i=1}^{n} x_i \cdot p(x_i)$, for the expected value of a probability distribution?	*Approaching*	*On Target*	*Working to Exceed*
#2	KU	i) Consider what each aspect of the formula represents and use them accordingly to explain? ii) Discuss the criteria that specify a binomial distribution?	*Approaching*	*On Target*	*Working to Exceed*
#3	KU	Consider what is known about the number of trials in a geometric distribution?	*Approaching*	*On Target*	

Part B-Application (APP)

Answer each of the problems that follow. If using Onenote Classroom Notebook, answer between each problem; if completing by hand, do so on lined paper and be prepared to hand in your assignment sheet with your completed assignment. Check the success criteria, below this problem set, for hints as to what you should try to emphasize in your solutions.

Problems

1. A game is played by drawing cards from a deck that has all the face cards removed including the aces. The player draws a card and is paid the face value of the card in dollars. What is the expected value of this game?

2. The probability of recovering after a particular type of experimental surgery is 0.4. If 3 patients undergo this operation, ...

a) Complete the binomial probability distribution table for the number of recovering patients. In this case, S denotes recovery.

# of successes	Probability Expression	Value
FFF	$\binom{3}{0}0.4^0 0.6^3$	0.216
SFF	$\binom{3}{1}0.4^1 0.6^2$	0.432
SSF	$\binom{3}{2}0.4^2 0.6^1$	0.288
SSS	$\binom{3}{3}0.4^3 0.6^0$	0.064

$\Sigma p(x) = 1$

b) Determine the expected number of recoveries for this type of experimental surgery.

3. If the distribution in #2 were geometric (i.e., with an unspecified number of people), what would be the expected number of surgeries required before experiencing success (i.e., before experiencing a recovery)?

4. Suppose that 1 out of 50 cards in a scratch-and-win promotion gives a prize.

a) What is the probability of winning on your fourth try?

b) What is the expected number of cards you would have to try before winning?

Part B-Success Criteria

Before submitting your completed assignment, ensure that you have considered the following:

Did I …

	Category	Criteria			
#1	APP	Use (correctly) the formula, $P(X = x_i) = \sum_{i=1}^{n} x_i \cdot p(x_i)$, for the expected value of a probability distribution?	*Approaching*	*On Target*	*Working to Exceed*
#2	APP	Consider how I might use patterning to complete the column "Probability Expression"? And correctly evaluate each expression?	*Approaching*	*On Target*	
#3	APP	Correctly use the formula for calculating the expected value of a geometric distribution?	*Approaching*	*On Target*	
#4	APP	-Consider the nature of this distribution—binomial or geometric (and justify my choice of formula(s))? -Use appropriate formula correctly?	*Approaching*	*On Target*	*Working to Exceed*

MDM 4U ASSIGNMENT #4 SOLUTIONS

PART A

#1. $$E(X) = \sum_{i=1}^{5} x_i \cdot p(X = x_i)$$

$$= 0 \cdot P(X=0) + 1 \cdot P(X=1) + \ldots + 4 \cdot P(X=4)$$

$$= 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16}$$

$$= 0 + \frac{4}{16} + \frac{12}{16} + \frac{12}{16} + \frac{4}{16}$$

$$= \frac{32}{16}$$

$$= 2$$

∴ The expected number of heads is 2.

#2 i) a) $\binom{20}{k}\left(\frac{1}{6}\right)^{k}\left(\frac{5}{6}\right)^{20-k}$

ii)
- Two outcomes : success (rolling a 3), failure (not rolling 3)
- Independent Trials
- Identical Trials
- Number of trials specified

#3. Given that the number of trials (i.e., 100) is specified, this problem cannot be modeled with a geometric distribution.

Rewriting the problem, as such, would allow for modeling by a geometric distribution:

"What is the expected number of failures for launching a rocket that has a failure rate of 1.5 %?"

Part B.

1. Let X be the event of drawing a card from the following set of values : { 2's, 3's, ..., 9's, 10's }.

For this set, there a 36 possible cards. And for any one value, there are 4 cards. This means that the probability of drawing any particular numbered-card is

$\frac{4}{36}$ or $\frac{1}{9}$.

X	2	3	...	9	10
P(X)	$\frac{1}{9}$	$\frac{1}{9}$	...	$\frac{1}{9}$	$\frac{1}{9}$

$$E(X) = 2 \cdot P(X=2) + 3 \cdot P(X=3) + \dots + 9 \cdot P(X=9) + 10 \cdot P(X=10)$$

$$= 2\left(\tfrac{1}{9}\right) + 3\left(\tfrac{1}{9}\right) + \dots 9\left(\tfrac{1}{9}\right) + 10\left(\tfrac{1}{9}\right)$$

$$= \frac{1}{9}(2 + 3 + \dots + 9 + 10)$$

$$= \frac{1}{9}(54)$$

$$= 6$$

∴ The expected value (i.e., payout) of this game is $6.

2b) $E(X) = np$

$= 3(0.4)$

$= 1.2$

∴ 1.2 (out of 3) patients are expected to recover.

#3. The expectation for a geometric distribution is given by

$$E(x) = \frac{q}{p}.$$

With $p = 0.4$ and $q = 1 - 0.4 = 0.6$, ...

$$E(x) = \frac{0.6}{0.4}$$

$$= \frac{3}{2} \text{ or } 1.5$$

∴ The expected number of recoveries is 1.5 patients.

#4. Given the geometric nature of the distribution and having to experience $x = 3$ failures before winning on the fourth try, ...

$$P(X = 3) = \left(\frac{49}{50}\right)^3 \left(\frac{1}{50}\right) \equiv P(X = x) = q^x p$$

$$= 0.98^3 \cdot 0.02$$

$$\doteq 0.019$$

∴ The probability of winning on your fourth try is about 1.9%.