Probability Distributions

Recall that a probability distribution is a table, formula, and/or graph that gives the probabilities for all possible values of a discrete random variable.

Probability Distributions (Types)

- Binomial Distributions
- Geometric Distributions
  --> Hypergeometric Distributions
- Normal Distributions

Binomial Distributions

When you write a test, there is generally one answer to the problem in question--it's either correct or incorrect.

- In this context, the outcomes of being correct or incorrect form a binomial event.
- The outcomes arise as a result of Bernoulli trials--independent trials that have two possible outcomes (correct = 'success'; incorrect = 'failure').
- Bernoulli trials are a property of binomial experiments.

E.g., Binomial or Not?

Which of the following can be represented by a binomial distribution?

a) getting tails 4 times on 10 tosses of a coin

Yes. 2 possible outcomes. Each of the 10 experiments are exactly the same, and the result of any one toss does not depend on the result of another--they are independent and identical.

b) rolling three 5's out of 15 rolls of a 6-sided die

Yes. There are six outcomes, but we're only interested in getting a result of 5 or not 5. This is an experiment with 2 results. Each of the 15 rolls is identical and independent of any other roll.

c) rolling three 5's or three 2's out of 15 rolls of a die

No. There are three possible outcomes--a 5, 2, or another number.

E.g., Success or Failure?
Calculate the probabilities for success and failure on each trial in the following binomial experiments.

a) getting tails 4 times on 10 tosses of a coin

-possible outcomes are H and T
-probability of success, p, is 1/2
-probability of failure is q = 1 - p = 1 -(1/2) = 1/2

b) rolling three 5's out of 15 rolls of a six-sided die

-success is rolling a 5; failure, not rolling a 5
-probabilities are not equal as in a)
-probability of success, p = 1/6
-probability of failure, q = 1 - (1/6) = 5/6

Determining a Binomial Distribution

E.g., 1. Rolling 5's

a) A die is rolled 3 times. What is the probability that the first roll is a 5?

Roll 1	Roll 2	Roll 3
5	?	?

$P(roll\,1 = 5) = \frac{1}{6}$ $P(roll\ 2 \neq 5) = \frac{5}{6}$ $P(roll\ 3 \neq 5) = \frac{5}{6}$

$$P(1st\ roll = 5) = P(roll\,1 = 5)\ AND\ P(roll\,2 \neq 5)\ AND\ P(roll\,3 \neq 5)$$
$$= \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$$
$$= \left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2$$

b) Determine the probability that the roll of 5 will appear in any of the three available positions in the table.

Roll 1	Roll 2	Roll 3
?	?	?

The number of ways that a single 5 can be placed into the table is the same as th number of combinations of 3 items choosing 1 at a time. Thus, there are

$\binom{3}{1}$ ways of placing the 5 into the table.

$$P(one\,5\,in\,three\,trials) = \binom{3}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2$$

b) Determine the probability that exactly two 5's show in 3 rolls of the die. For example, let's say that one possible outcome is 'Roll 1 = 5, Roll 2 = 5, Roll 3 = anythir but a 5.

Roll 1	Roll 2	Roll 3
5	5	?

It follows that $P(Roll1 = 5 \cap Roll2 = 5 \cap Roll3 \neq 5)$

$$= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$$

$$= \left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1$$

The number of ways that a two 5's can be placed into the table is the same as the number of combinations of 3 items choosing 2 at a time. Thus, there are

$\binom{3}{2}$ ways of placing the 5's into the table.

$$P(two\,5s\,three\,trials) = \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1$$

d) Up to this point in time, you have been 'secretly' working to complete the probability distribution for this binomial distribution.

Use your knowledge from parts b) and c) to complete the table (below) for this distribution.

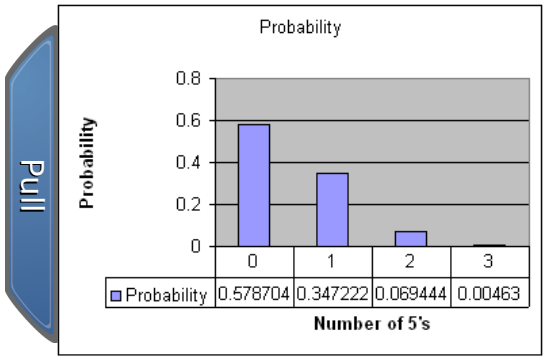
Number of 5's	Probability
0	
1	
2	
3	

d) Up to this point in time, you have been 'secretly' working to complete the probability distribution for this binomial distribution.

Use your knowledge from parts b) and c) to complete the table (below) for this distribution.

Number of 5's	Probability
0	$\binom{3}{0}\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{3}$
1	$\binom{3}{1}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{2}$
2	$\binom{3}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{1}$
3	$\binom{3}{3}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{0}$

e) Sketch the graph of the probability distribution you've determined from the table.



f) Write a formula for P(X = r), where X is the discrete random variable that corresponds to the number of successes, for this probability distribution.