Probability Distributions

Recall that a *probability distribution* is a table, formula, and/or graph that gives the probabilities for all possible values of a discrete random variable.

Probability Distributions (Types)

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Binomial Distributions

When you write a test, there is generally one answer to 'closed' problems--correct or incorrect.

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E.g., Binomial or Not?

Which of the following can be represented by a binomial distribution?

a) getting tails 4 times on 10 tosses of a coin

b) rolling three 5's out of 15 rolls of a 6-sided die

c) rolling three 5's or three 2's out of 15 rolls of a die

E.g., Success or Failure?

Calculate the probabilities for success and failure on each trial in the following binomial experiments.

a) getting tails 4 times on 10 tosses of a coin

b) rolling three 5's out of 15 rolls of a six-sided die

Determining a Binomial Distribution

E.g., 1. Rolling 5's

a) A die is rolled 3 times. What is the probability that the first roll is a 5?

Roll 1	Roll 2	Roll 3
5	?	?

$P(roll\,1 = 5) = -$ $P(roll\ 2 \neq 5) = -$ $P(roll\ 3 \neq 5) = -$

b) Determine the probability that the roll of 5 will appear in any of the three available positions in the table.

Roll 1	Roll 2	Roll 3
?	?	?

c) Determine the probability that exactly two 5's show in 3 rolls of the die. For example, let's say that one possible outcome is 'Roll 1 = 5, Roll 2 = 5, Roll 3 = anything but a 5.

Roll 1	Roll 2	Roll 3
5	5	?

It follows that $P(Roll1 = 5 \cap Roll2 = 5 \cap Roll3 \neq 5)$

d) Up to this point in time, you have been 'secretly' working to complete the probability distribution for this binomial distribution.

Use your knowledge from parts b) and c) to complete the table (below) for this distribution.

Number of 5's	Probability
0	
1	
2	
3	

d) Up to this point in time, you have been 'secretly' working to complete the probability distribution for this binomial distribution.

Use your knowledge from parts b) and c) to complete the table (below) for this distribution.

Number of 5's	Probability
0	
1	
2	
3	

e) Sketch the graph of the probability distribution you've determined from the table.

Pull

f) Write a formula for $P(X = r)$, where X is the discrete random variable that corresponds to the number of successes, r, for this probability distribution.

$$P(X = r) = \binom{3}{r}\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{3-r}$$

Binomial Probability Distribution

Consider a binomial experiment in which there are n Bernoulli trials, each with a probability of success of p. The probability of k successes in the n trials is given by

$$P(X = k) = \binom{n}{k}(p)^k(1 - p)^{n-k}$$

where X is the discrete random variable corresponding to the number of successes.

Expected Value of a Binomial Experiment

The expected value of a binomial experiment that consists of n Bernoulli trials with a probability of success, p, on each trial is

$$E(X) = np$$

OR

expected value—the sum of the terms of the form $X \cdot P(X)$ for all possible values of a discrete random variable, X

$$E(X) = x_1P(X = x_1) + x_2P(X = x_2) + \ldots + x_nP(X = x_n)$$