Solution to Introductory Problem

Since $n = 8$ and $r = 3$ the number of combinations is given by

$$_8C_3 \text{ or } \binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8\times7\times6\times\cancel{5!}}{3!\,\cancel{5!}} = \frac{8\times7\times\cancel{6}}{\cancel{6}} = 8\times7 = 56$$

Cancel 5! from both numerator and denominator

Therefore, there are 56 combinations of 8 objects taken 3 at a time.

E.g., 2. How many different sampler dishes, with 3 different flavours, could you get at an ice-cream shop with 31 different flavours?

$$\binom{31}{3} = \frac{31!}{3!(31-3)!} = \frac{31\times30\times29\times\cancel{28!}}{3!\,\cancel{28!}} = \frac{31\times\cancel{30}^{5}\times29}{\cancel{6}} = 31\times29\times5 = 4495$$

E.g., 2. From a class of 19 students, determine how many ways a 5-person group can be selected to organize a class party

a) with no restrictions.

$$\binom{19}{5} = \frac{19!}{5!(19-5)!} = \frac{19!}{5!\,14!} = \frac{19\times18\times17\times16\times15\times\cancel{14!}}{5!\,\cancel{14!}} = \frac{19\times\cancel{18}^{\cancel{9}^{3}}\times17\times\cancel{16}^{4}\times\cancel{15}^{3}}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1} = \frac{19\times3\times17\times4\times3}{1} = 11628$$

b) with you on the committee.

$$\binom{19-1}{5-1} = \binom{18}{4}$$

E.g. 3.

a) In a standard deck of cards, how many 5-card hands will have 3 aces **and** 2 kings?

Solution: This solution not only involves combinations, but also . . . multiplicative counting principle

$$n(3A \cap 2K) = \binom{4}{3} \times \binom{4}{2}$$
$$= 4 \times 6$$
$$= 24$$

$$\frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4!}{4} = 6$$

b) How many 5-card hands will have 3 aces **or** 2 kings?

add

$$n(3A \cup 2K) = 4 + 6$$
$$= 10$$

E.g. 4. A delegation of three people is to be chosen from a group of community volunteers consisting of 4 lawyers, a doctor, and three teachers. In how many ways can this group be formed if at least one teacher must be a member of the delegation?

Solution:

Considering two forms of reasoning, direct or indirect, we can solve this problem. If we handle the problem directly, all suitable outcomes are totalled to get a final answer. We will consider three cases:

1. a delegation with one teacher;
2. a delegation with two teachers; **or**
3. a delegation with three teachers.

Case 1: $\underset{\text{Teacher}}{\binom{3}{1}} \times \overset{8-3}{\binom{}{2}} = \binom{3}{1} \times \binom{5}{2}$
$$= 3 \times 10$$
$$= 30$$

$$\frac{5!}{2!3!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2!}\,\cancel{3!}} = 5 \times 2 = 10$$

Case 2: $\binom{3}{2} \times \binom{5}{1} = 3 \times 5$
$$= 15$$

Case 3: $\binom{3}{3} \times \binom{5}{0} = 1 \times 1$
$$= 1$$

$$n(\text{at least one teacher}) = 30 + 15 + 1$$
$$= 46$$

Case 3:

If we handle the problem **indirectly**, all undesired outcomes are subtracted from the total to get the final answer. We will find the total number of three person delegations, without restrictions, and then subtract the number of delegations with no ~~retailers~~. teachers

Solution:

$$n(3\text{-person delegations}) = \binom{8}{3}$$

$$= 56$$

← already calculated in introductory example.

$$n(\text{no teachers}) = \binom{3}{0} \times \binom{5}{3}$$

$$= 1 \times 10$$

$$= 10$$

$$\therefore n(\text{at least 1 teacher}) = 56 - 10$$

$$= 46$$

$$\binom{5}{3} = \frac{5!}{3!(5-3)!}$$

$$= \frac{5 \times 4 \times \cancel{3!}}{2!\cancel{3!}}$$

$$= \frac{20}{2}$$

$$= 10$$