In the permutations section, we dealt with the number of ways in which we could choose an ordered arrangement of r elements from a set of n elements. In various applications of Combinatorics, we will not need to choose an ordered arrangement. When different orders are not considered to be distinct, we count the number of possible outcomes by using **combinations**. Different orders are not considered to be distinct (or considered to be the same outcome). For instance, by disregarding order, (A, B) = (B, A).

Introductory Problem:

Suppose an art museum owns 8 paintings by a given artist and another art museum wishes to borrow 3 of these paintings for a special show. In selecting 3 of the 8 paintings for shipment, the order would not matter, and we would simply be selecting a 3-element subset from the set of 8 paintings. That is, we would be selecting what is called a **combination of 8 objects taken 3 at a time**.

Definition: A Combination of *n* Objects Taken *r* at a Time
A combination of a set of n distinct objects taken r at a time, without repetition, is an r-element subset of the set of n objects. The arrangement of the elements in the subset does not matter.

Definition: Number of Combinations of *n* Objects Taken *r* at a Time

The number of combinations of n distinct objects taken r at a time, without repetition, is given by

$$\begin{aligned} C(n,r) &= \binom{n}{r} \\ &= \frac{P(n,r)}{r!} \\ &= \frac{n!}{r!(n-r)!},\ 0 \le r \le n \end{aligned}$$

Note $C(n,r)$, $\binom{n}{r}$, nCr, *and* C_r^n all mean the same thing. Each notation is read as "n choose r."

Solution to Introductory Problem

Since $n = 8$ and $r = 3$ the number of combinations is given by

Therefore, there are ____ combinations of 8 objects taken 3 at a time.

E.g., 2. How many different sampler dishes, with 3 different flavours, could you get at an ice-cream shop with 31 different flavours?

E.g., 3. From a class of ____ students, determine how many ways a 5-person group can be selected to organize a class party

a) with no restrictions.

b) with you on the committee.

E.g. 4.
a) In a standard deck of cards, how many 5-card hands will have 3 aces **and** 2 kings?

Solution: This solution not only involves combinations, but also . . .

b) How many 5-card hands will have 3 aces **or** 2 kings?

E.g., 5. A delegation of three people is to be chosen from a group of community volunteers consisting of 4 lawyers, a doctor, and three teachers. In how many ways can this group be formed if at least one teacher must be a member of the delegation?

Solution:
Considering two forms of reasoning, direct or indirect, we can solve this problem. If we handle the problem directly, all suitable outcomes are totalled to get a final answer. We will consider three cases:

1. a delegation with one teacher;
2. a delegation with two teachers; **or**
3. a delegation with three teachers.

Case 1:

Case 2:

Case 3:

If we handle the problem **indirectly**, all undesired outcomes are subtracted from the total to get the final answer. We will find the total number of three person delegations, without restrictions, and then subtract the number of delegations with no retailers.

Solution:

E.g., 6. Solve the equation $4\binom{n}{2} = \binom{n+2}{3}$ for $n \in N$.