Compound Interest

Learning Goal

Minds on Math...Compound Interest Formula

Consider the following statement:

*A student puts **$1000** of their earnings into an investment that pays 4.8% (i.e., **0.048**) per year, compounded annually (i.e., interest is paid once at the end of each year) for **3** years. How much will their investment be worth at the end of 3 years?*

Drag the values from above into the compound interest formula below.

$1000

0.048

? $\rightarrow$ $$A = P(1 + i)^n$$ $\leftarrow$ 3

Check

Minds on Math...Calculator Operations

$1000 **0.048**

Check **?** $A = P(1 + i)^n$ **3**

Organize the following calculator keystrokes to produce the correct answer of **$1151.02**.

Drag in

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7
1.048	y^x	3	=	x	$1000	=

= 3 x = 1.048 y^x $1000

Minds on Math...The Mathematical Solution

A student puts ***$1000*** *of their earnings into an investment that pays 4.8% (i.e.,* ***0.048****) per year, compounded annually (i.e., interest is paid once at the end of each year) for* ***3*** *years. How much will their investment be worth at the end of 3 years****?***

Solution:

$$A = P(1 + i)^n$$

$$= \$1000(1.048)^3$$

$$= \$1151.02$$

Therefore, in 3 years, there will be $1151.02 in the account.

Pull

Minds on Math...Detecting Compounding Frequencies

What's the relationship of $1000(1.048)³ to the remaining expressions for the amount, *A*, of an investment (or loan)?

	Relationship to $1000(1+0.048)³	Compounding Frequency
$1000(1+0.024)⁶		
$1000(1+0.012)¹²		
$1000(1+0.004)³⁶		
$1000(1+0.002)⁷²		

Reflection: Imagine how the statement would read using the various compounding frequencies (see table).

*A student puts **$1000** of their earnings into an investment that pays 4.8% (i.e., **0.048**) per year, compounded annually (i.e., interest is paid once at the end of each year) for **3** years. How much will their investment be worth at the end of 3 years?*

Minds on Math...Detecting Compounding Frequencies

What's the relationship of $1000(1.048)³ to the remaining expressions for the amount, *A*, of an investment (or loan)?

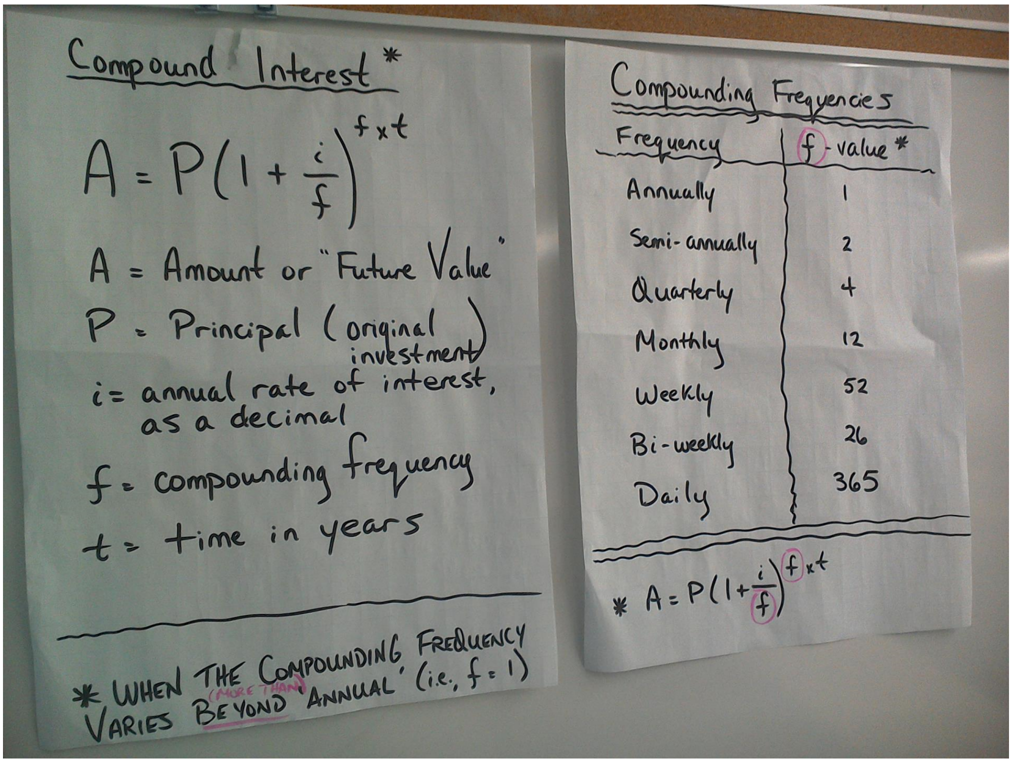
	Relationship to $1000(1+0.048)³	Compounding Frequency
$1000(1+0.024)⁶	$\$1000\left(1+\frac{0.048}{2}\right)^{3\times 2}$	semi-annually; twice per year
$1000(1+0.012)¹²	$\$1000\left(1+\frac{0.048}{4}\right)^{3\times 4}$	quarterly; four times per year
$1000(1+0.004)³⁶	$\$1000\left(1+\frac{0.048}{12}\right)^{3\times 12}$	monthly; every month
$1000(1+0.002)⁷²	$\$1000\left(1+\frac{0.048}{24}\right)^{3\times 24}$	semi-monthly; twice per month

Reflection: Imagine how the statement would read using the various compounding frequencies (see table).

*A student puts **$1000** of their earnings into an investment that pays 4.8% (i.e., **0.048**) per year, compounded annually (i.e., interest is paid once at the end of each year) for **3** years. How much will their investment be worth at the end of 3 years?*

Consolidate: Compound Interest Formula & Compounding Frequencies

Key Ideas



Take Action: Example 1-Comparing Plans

Problem

You're planning on borrowing $3000. You have two options:

Option 1: $3000 for five years at 9% compounded semi-annually

Option 2: $3000 for five years at 8.6% per year, compounded quarterly

Which option will you choose? Justify your choice.

$A = \$3000\left(1 + \frac{0.09}{2}\right)^{5\times 2}$

$= \$3000(1.045)^{10}$

$= \$4658.91$

$A = \$3000\left(1 + \frac{0.086}{4}\right)^{5\times 4}$

$= \$3000(1.0215)^{20}$

$= \$4590.80$

Conclusion: Go with Option 2; it produces ~~more~~ less interest.

Take Action: Example 2-Doubling Time

Problem

How long will it take a sum of money to double at 4.8% per year, compounded *annually*?

Solution

n	$\$1(1.048)^n$	>, <, = to \$2
10		
15		

Conclusion

Therefore, it takes ____ years for money to double at 4.8% compounded annually.

Take Action: Example 2-Doubling Time

Problem

How long will it take a sum of money to double at 4.8% per year, compounded *annually*?

Solution

Let's say that P = \$1. This means that the future value must be A = \$2.

So the compound interest equation is written as $2 = 1(1.048)^n$.

n	$\$1(1.048)^n$	>, <, = to \$2
10	1.60	<
15	2.02	>
14	1.92	<
14.5	1.97	<
14.8	2.00	=

Conclusion

Therefore, it takes 14.8 years for money to double at 4.8% compounded annually.

Consolidate Your Understanding: PRACTICE

p432 #2

p433 #6

p434 #10, 14

} due: tomorrow.