Conditional Probability

Minds on Math: Classifying Compound Events

Definition:

Compound events are made up of two or more simple events. The following are examples of compound events:

a) flipping a coin and then rolling a die to see if you get heads (or tails) and ___ (#)

b) flipping a coin three times to see if you get ____ (#) heads (or tails) in a row

c) drawing two cards from a deck, one at a time, to see if you get two _____

Problem 1
Study the problems, above, such that you are able to classify them according to your own criteria.

Minds on Math (contd.)

Summary

- The events in a)...are independent
- The events in b)...are independent
- The events in c)...are dependent

Recall

In general, when a compound event occurs, its probability is the product of the individual simple event probabilities. The formula below calculates the probability of independent events.

$$P(A \cap B) = P(A) \times P(B)$$

Problem 2

Solve the problem in c) where you're trying to determine the probability of drawing two aces in a row (assume that the first card is not returned to the deck). Space, for your solution, has been provided on the next page.

Solution to c):

Let A be the event that the first card drawn is an ace; B, the second card drawn is also an ace.

(Note that the goal of the problem is to find the probability that both the first AND second cards drawn are aces--i.e., determine P(A and B).)

As per the multiplicative rule for independent events,

$$P(A \cap B) = P(A) \times P(B)$$

In this example, P(A) = 4/52. Thus,

$$P(A \cap B) = \frac{4}{52} \times P(B)$$

By this line of reasoning, P(B), *given that A has already occurred*, is given by

$$P(B \text{ given } A) = \frac{3}{51}$$

Carrying these ratios forward, the following probability is obtained:

$$P(A \cap B) = P(A) \times P(B \text{ given } A)$$
$$= \frac{4}{52} \times \frac{3}{51}$$
$$= \frac{1}{13} \times \frac{1}{17}$$
$$= \frac{1}{221}$$

Therefore, the probability of drawing two aces in a row, given that the first card drawn is not replaced, is 1/221.

Solution to c):

Let A be the event that the first card drawn is an ace; B, the second card drawn is also an ace.

(Note that the goal of the problem is to find the probability that both the first AND second cards drawn are aces--i.e., determine P(A and B).)

As per the multiplicative rule for independent events,

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In this example, P(A) = 4/52. Thus,

$$P(A \cap B) = \frac{4}{52} \times P(B)$$

But, the outcome of event B *is dependent upon* the outcome of event A. Thus, when calculating the probability of B, we must restrict the sample space to that of B only, where there are only 51 cards--not 52.

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Carrying these ratios forward, the following probability is obtained:

$$P(A \cap B) = P(A) \times P(B \text{ given } A)$$
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Therefore, the probability of drawing two aces in a row, given that the first card drawn is not replaced, is 1/221.

The formula you've applied, above, is used to determine the probability of compound events where there is a condition. The formula for determining conditional probability is given by

$$P(A \cap B) = P(A) \times P(B \text{ given } A)$$
$$= P(A) \times P(B \mid A)$$

E.g., The probability that Pat will go to U of T is 1/5. The probability that she will go to another university is 1/2. If Pat goes to U of T, the probability that she will get financial aid is 3/4. What is the probability that Pat will go to U of T and get financial aid?

Solution:

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Solution:

$$P(go \cap aid) = P(go) \times P(aid \mid go)$$
$$= \frac{1}{5} \times \frac{3}{4}$$
$$= \frac{3}{20}$$

Therefore, Pat has a 3/20 chance of going to U of T and receiving financial aid.

Exit Problems_4.4_Cond Prob

1 A class is surveyed to determine whether they prefer mathematics or english. The table shows the results. State P(male|prefers english).

Gender	Mathematics	English
Males	4	9
Females	7	8

A $\frac{9}{17}$

B $\frac{9}{13}$

C $\frac{9}{4}$

D none of the above

2 Determine the probability of drawing a spade and then a club from a regular deck of cards if the spade is *not returned* to the deck.

A $\frac{1}{16}$

B $\frac{1}{2}$

C $\frac{3}{51}$

D $\frac{13}{204}$

3 A pair of students is picked randomly from four students John, Sara, Adam, and Laura. Determine the probability that a girl will be chosen *given* that Adam has been chosen already.

A $\frac{1}{6}$

B $\frac{2}{3}$

C $\frac{1}{3}$

D $\frac{1}{2}$