p124 #9

C(20, -21) and D(-20, 21) are the endpoints of the diameter of the circle.

By determining the length of the diameter, the length of the radius can be found since

$d = 2r \; ; \; \frac{d}{2} = r$

Using the distance formula, $d = \sqrt{\Delta x^2 + \Delta y^2}$,

$$\begin{aligned} d &= \sqrt{(20-(-20))^2 + (-21-21)^2} \\ &= \sqrt{(20+20)^2 + (-42)^2} \\ &= \sqrt{40^2 + (-42)^2} \\ &= \sqrt{1600 + 1764} \\ &= \sqrt{3364} \\ &= 58 \end{aligned}$$

$\therefore \; d = 58 \; , \; r = \frac{58}{2} = 29$

The equation of a circle is given by

$x^2 + y^2 = r^2$ where r = radius.

$\therefore \; x^2 + y^2 = 29^2 \; ; \; x^2 + y^2 = 841$

is the equation of the circle.

A (1)

p 124 #11

The point $(-2, k)$ lies on the circumference of the circle $x^2 + y^2 = 20$. Determine the values of k.

Since $(-2, k)$ lies on the circle, its coordinates satisfy the equation of the circle.

In the equation, $x^2 + y^2 = 20$, set $x = -2$ and $y = k$.

$$(-2)^2 + k^2 = 20$$

Solving for k, ...

$$4 + k^2 = 20$$

$$k^2 = 20 - 4$$

$$k^2 = 16$$

$$\sqrt{k^2} = \sqrt{16}$$

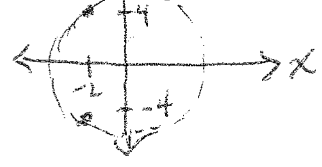
Since the square root of a variable expression returns two roots – a positive and a negative root — ...

$$\sqrt{k^2} = \pm\sqrt{16}$$

$$k = \pm 4$$

Note: This solution is consistent with what we know about circles and the symmetry of points with like x- or y-coordinates.

i.e.,



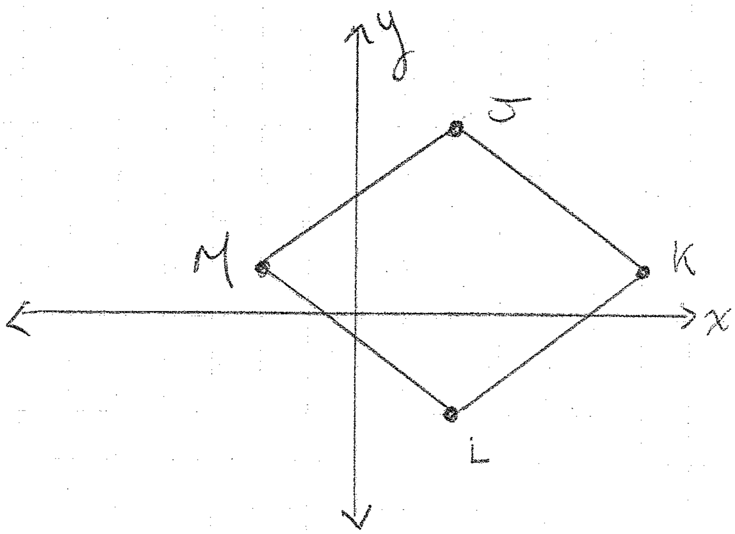
Both $(-2, 4)$ and $(-2, -4)$ are on the circumference of the circle.

A 2

p126 # 5, 6 OR 8

#5. JKLM has vertices at J(2,4), K(6,1), L(2,-2), and M(-2,1).

To determine the type of quadrilateral, first start by plotting the coordinates of each ordered pair.



JKLM has the appearance of a rhombus.

Rhombii have equal side lengths and opposite sides that are parallel.

Prove: JKLM is a rhombus.

Proof:

Side lengths:

$$
\left.\begin{aligned}
|JK| &= \sqrt{(2-6)^2+(4-1)^2} \\
&= \sqrt{(-4)^2+3^2} \\
|ML| &= \sqrt{(-2-2)^2+(1-(-2))^2} \\
&= \sqrt{(-4)^2+3^2}
\end{aligned}\right\} \text{Corresponding side lengths}
$$

From the above, $|JK| = |ML|$

In like manner, ...

$$
\begin{aligned}
|JM| &= \sqrt{(2-(-2))^2+(4-1)^2} \\
&= \sqrt{4^2+3^2} \\
&\equiv \sqrt{(-4)^2+3^2}
\end{aligned}
\quad \text{AND} \quad
\begin{aligned}
|KL| &= \sqrt{(6-2)^2+(1-(-2))^2} \\
&= \sqrt{4^2+3^2} \\
&\equiv \sqrt{(-4)^2+3^2}
\end{aligned}
$$

$\therefore$ Altogether $|JK| = |ML| = |JM| = |KL|$

3 1

Slopes:

$$m_{JK} = \frac{4-1}{2-6} = \frac{3}{-4}$$

$$m_{JM} = \frac{4-1}{2-(-2)} = \frac{3}{4}$$

$$m_{ML} = \frac{1-(-2)}{-2-2} = \frac{3}{-4}$$

$$m_{KL} = \frac{1-(-2)}{6-2} = \frac{3}{4}$$

From the above $m_{JK} = m_{ML}$; thus $JK \parallel ML$.

$m_{JM} = m_{KL}$; thus $JM \parallel KL$.

$\therefore$ All sides are equal in length and opposite sides are parallel, JKLM is a rhombus.

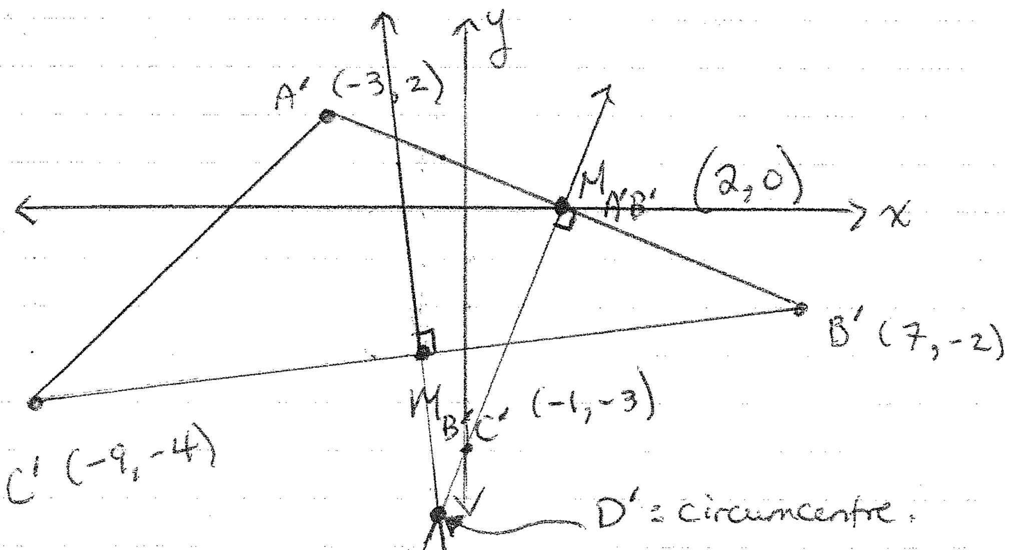
B(2)

p 126 #6

Since the new fountain is equidistant from the vertices of $\triangle ABC$, it is located at the triangle's circumcentre — that is, at the intersection of its perpendicular bisectors.

Using a scale of 8 : 1, each set of coordinates maps as follows :

$A(-24, 16) \longrightarrow A'(-3, 2)$
$B(56, -16) \longrightarrow B'(7, -2)$
$C(-72, -32) \longrightarrow C'(-9, -4)$



To find the intersection of right bisectors, the equations of any two right bisectors will need to be found.

Right Bisector from B'C':

To find the right bisector from B'C', find $-\dfrac{1}{m_{B'C'}}$

and the midpoint of B'C', $M_{B'C'}$.

negative reciprocal of $m_{B'C'}$.

p126 #6 (Cont'd.)

$$m_{B'C'} = \frac{-2-(-4)}{7-(-9)}$$
$$= \frac{-2+4}{7+9}$$
$$= \frac{2}{16}$$
$$= \frac{1}{8}$$

Thus, $-\frac{1}{m_{B'C'}} = -\frac{8}{1}$ or -8

∴ The slope of the right bisector from B'C' is −8.

To determine the y-intercept of its equation, determine the midpoint of B'C', substitute its values in to $y = -8x + b$, and solve for b.

$$M_{B'C'} = \left(\frac{7+(-9)}{2}, \frac{-2+(-4)}{2}\right)$$
$$= (-1, -3)$$

Equation: $y = -8x + b$

$$-3 = -8(-1) + b$$
$$-3 = 8 + b$$
$$-11 = b$$

∴ The equation of the right bisector from B'C' is $y = -8x - 11$.

Repeat this process again for another right bisector.

For the right bisector from A'B':

$$m_{A'B'} = \frac{2-(-2)}{-3-7}$$
$$= \frac{4}{-10}$$
$$= -\frac{2}{5}$$

$$-\frac{1}{m_{A'B'}} = \frac{5}{2}$$

∴ Slope of the right bisector from A'B' is $\frac{5}{2}$.

$$M_{A'B'} = (2, 0)$$

Equation: $y = \frac{5}{2}x + b$

$$0 = \frac{5}{2}(2) + b$$
$$0 = 5 + b$$
$$-5 = b$$

∴ The equation of the right bisector from A'B' is $y = \frac{5}{2}x - 5$

C2

p126 #6 (contd.)

Given that $\triangle ABC$ is obtuse, the circumcentre lies outside of the triangle. This can be seen in the graph.

To find the intersection of both right bisectors, use either elimination or substitution.

$$y = -8x - 11 \quad (1)$$

$$y = \frac{5}{2}x - 5 \quad (2)$$

$$0 = -8x - \frac{5}{2}x - 11 - (-5) \quad (1) - (2)$$

$$0 = \frac{-16}{2}x - \frac{5}{2}x - 11 + 5$$

$$0 = \frac{-21}{2}x - 6$$

$$\frac{21}{2}x = -6$$

$$\frac{2}{21}\left(\frac{21}{2}x\right) = -6\left(\frac{2}{21}\right)$$

$$x = -\frac{12}{21}$$

$$x = -\frac{4}{7}$$

Set $x = -\frac{4}{7}$ in (2):

$$y = \frac{5}{2}\left(\frac{-4}{7}\right) - 5$$

$$= \frac{-20}{14} - 5 \quad \text{(to top right)}$$

$$y = \frac{-10}{7} - 5$$

$$= \frac{-10 - 35}{7}$$

$$= \frac{-45}{7}$$

$\therefore D'(x, y) = \left(\frac{-4}{7}, \frac{-45}{7}\right)$ is the POI — i.e., the circumcentre.

Lastly, scale up the solution using 8 : 1 as the scale factor.

$$D'(x, y) = \left(\frac{-4}{7}, \frac{-45}{7}\right) \rightarrow D\left(\frac{-4 \times 8}{7}, \frac{-45 \times}{7}\right.$$

$$\rightarrow D\left(\frac{-32}{7}, \frac{-36}{7}\right.$$

$\therefore$ The new fountain can be located at $\left(\frac{-32}{7}, \frac{-360}{7}\right)$.

C3

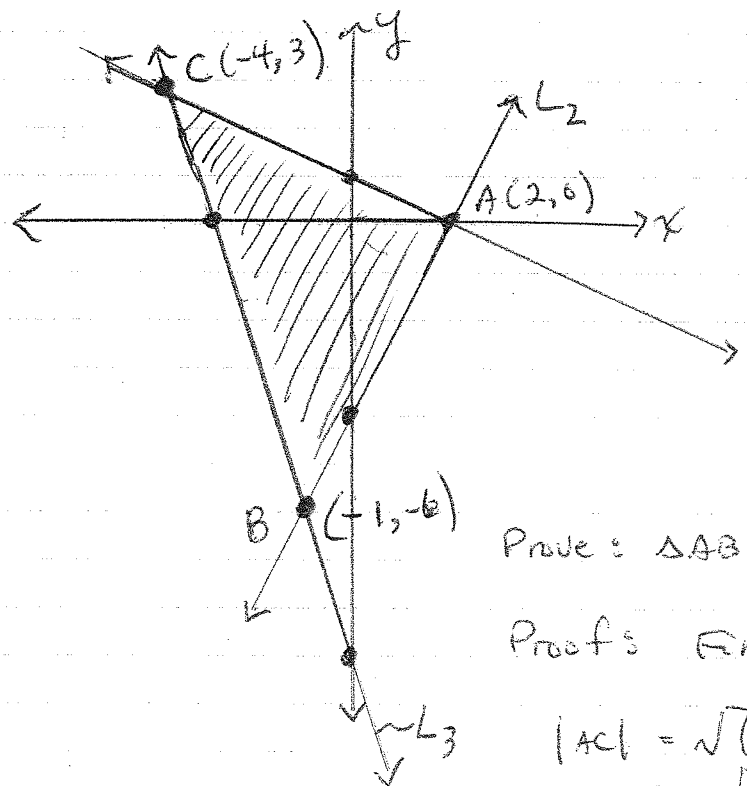
p 126 #8.

$L_1: x + 2y - 2 = 0 \iff x + 2y = 2$; x-int = 2, y-int = 1

$L_2: 2x - 4 - y = 0 \iff 2x - y = 4$; x-int = 2, y-int = −4

$L_3: 3x + y + 9 = 0 \iff 3x + y = -9$; x-int = −3, y-int = −9

Use the predetermined intercepts to graph the three lines.



$\triangle ABC$, with coordinates, $A(2,0)$, $B(-1,-6)$, and $C(-4,3)$ appears to be isosceles. If isosceles, then

$|AC| = |AB|$

Prove: $\triangle ABC$ is isosceles.

Proof: Find $|AC|$ and $|AB|$.

$$|AC| = \sqrt{(2-(-4))^2 + (0-3)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45}$$

$$|AB| = \sqrt{(2-(-1))^2 + (0-(-6))^2} = \sqrt{3^2 + 6^2} = \sqrt{45}$$

$\therefore |AC| = |AB|$, $\triangle ABC$ is isosceles.

①

p126 #8.

$\therefore \angle A > \angle B > \angle C$, $|BC| > |AC|$ and $|BC| > |AB|$,

So $\triangle ABC$ cannot be equilateral.

Note that $\angle BAC$ appears to be $90°$.

$\triangle ABC$ could be right isosceles.

Determine m_{AC} and m_{AB} to determine if

$m_{AC} = -\frac{1}{m_{AB}}$ (i.e., if they are negative reciprocals)

$m_{AC} \times m_{AB} = -1$ (negative reciprocals multiply to -1)

$m_{AC} = \frac{0-3}{2-(-4)}$; $m_{AB} = \frac{0-(-6)}{2-(-1)}$

$= \frac{-3}{6}$ $\qquad = \frac{6}{3}$

$= -\frac{1}{2}$ $\qquad = 2$

As can be seen $m_{AC} \times m_{AB} = -\frac{1}{2} \times 2$

$= -1$

$\therefore$ The slopes are negative reciprocals.
$\triangle ABC$ is also a right triangle.

In conclusion, $\triangle ABC$ is right isosceles.