MDM 4U

Probability Distributions and Expected Value

From the beginning of the course, an emphasis has been placed on the probability of individual outcomes from an experiment.

Moving forward, we will now use models for distributions that show the probabilities of all possible outcomes of an experiment.

For example, if 3 coins are tossed, we may be interested in the number of heads that turn up rather than in the particular pattern that turns up. In certain games involving dice, players are often interested in the sum of the number of dots that show up on two dice rather than the actual number on each.

Probability Distributions

To define a probability distribution, the probability of each outcome must be determined. The distribution of these probabilities may then be provided in the form of a graph or table.

E.g., 1. Create both a graphical and tabular probability distribution for the **discrete, random variable*** X. X is defined as the sum of the roll of two dice.

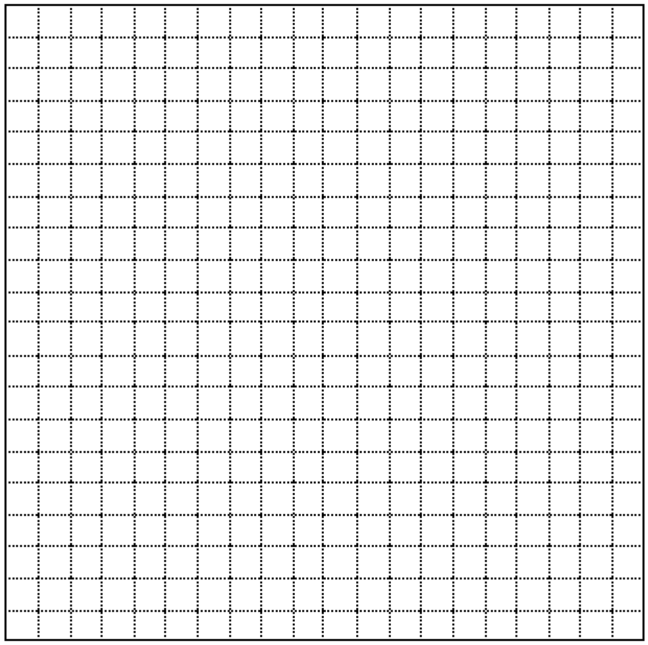
Die 1

Die 2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

*a variable that assumes a unique value for each outcome

The following represents the graphical model with probability (i.e., relative frequency) plotted against sum.



<u>Expected Value of a Discrete Random Variable</u>

Once a probability distribution has been defined**, then this model can be used to further analyze an experiment. A useful piece of information that can be obtained from the distribution is the **expected value** or **expectation**. This value represents the quantity that you can expect to obtain when an experiment is performed and is denoted by **E(x).** Let's examine this concept with the following example.

E.g., 2. A game is defined by the rules that two dice are rolled and the player wins varying amounts depending on the sum of the two dice according to the table below.

Sum	Winnings ($)	Sum	Winnings ($)
2	10	7	5
3	9	8	6
4	8	9	7
5	7	10	8
6	6	11	9
		12	10

a) What can a player expect to win by playing this game?
b) What would be a fair value to pay to play this game?

**numerical (table), graphical, algebraic

Solution:
a) Let the random variable X be the winnings associated with each roll.

Winnings (x)	Sum	Probability $p(x)$	$x(p(x))$
5			
6			
7			
8			
9			
10			

b) Fair value to play this game?

In the previous example, the expected value was calculated and defined by the following expression:

$$E(x) = \sum_{x} x \bullet p(x)$$

Below, you will find the expansion for the above function.

Expected Value of a Discrete Random Variable

The expected value of a discrete random variable, X, is the sum of the terms of the form $X \cdot P(X)$ for all possible values of X. In other words, if X takes on the values $x_1, x_2, \ldots, x_n$, then the expected value of X is given by

$$E(X) = x_1P(X = x_1) + x_2P(X = x_2) + \ldots + x_nP(X = x_n)$$
$$= \sum_{i=1}^{n} x_iP(X = x_i)$$

where n represents the number of terms in the sum.

Now, we will further explore the concept of expected value with a few examples.

E.g., 3. Suppose you were to toss three coins.

a) What is the likelihood that you would observe at least two heads?

b) What is the expected number of heads?

E.g., 4. A committee of four people is to be chosen randomly from four males and six females. What is the expected number of females on the committee?

Solution: To find the expectation, first determine the probability distribution. Let the random variable X be the number of females on the committee. In this situation, we know that $X = \{0, 1, 2, 3, 4\}$.

Tabulated results:

# of Females (x)	Probability $p(x)$	$x(p(x))$
0		
1		
2		
3		
4		