

Exponential Growth & Decay

Learning Goals

Minds on Math #1

Representation: 1

5, 9, 13, 17, 21, ...	5, 15, 45, 135, 405, ...
5, 2, -1, -4, -7, ...	10, 13, 16, 19, 22, ...

Which one doesn't belong?

5, 15, 45, 135, 405, ...

This sequence shows "exponential growth"

-the same ratio occurs between consecutive terms

$$\text{E.g., } \frac{15}{5} = 3 ; \frac{45}{15} = 3 ;$$
$$\frac{135}{45} = 3 ; \text{ and so on.}$$

Minds on Math #2 Representation: Table of Values

Which one doesn't belong?

Minds on Math #3 Representation: Table of Values

Which one doesn't belong?

A

x	y
-2	-4
-1	-1
0	2
1	5
2	8

B

x	y
0	0
1	1
2	4
3	9
4	16
5	25
6	36

C

x	y
0	4
1	12
2	36
3	108
4	324

x	y
0	0
1	1
2	4
3	9
4	16
5	25
6	36

difference of y-values

$$\begin{aligned} 1-0 &= 1 \\ 4-1 &= 3 \\ 9-4 &= 5 \\ 16-9 &= 7 \\ 25-16 &= 9 \\ 36-25 &= 11 \end{aligned}$$

difference of differences

$$\begin{aligned} 3-1 &= 2 \\ 5-3 &= 2 \\ 7-5 &= 2 \\ 9-7 &= 2 \\ 11-9 &= 2 \end{aligned}$$

B

x	y
0	4
1	12
2	36
3	108
4	324

ratio of y-values

$$\begin{aligned} \frac{12}{4} &= 3 \\ \frac{36}{12} &= 3 \\ \frac{108}{36} &= 3 \\ \frac{324}{108} &= 3 \end{aligned}$$

C

x	y
-2	-4
-1	-1
0	2
1	5
2	8

difference of y-values

$$\begin{aligned} -1+4 &= 3 \\ 2+1 &= 3 \\ 5-2 &= 3 \\ 8-5 &= 3 \end{aligned}$$

A

Consolidate
 Each of the representations demonstrate the same relationship(s) in a given set of data

?-Other: Equation
 E.g., $y = mx + b$ (linear)
 E.g., $y = ax^2 + bx + c$ (quadratic)
 E.g., $y = Ab^x$

?-Other: Pictorial
 1 2 3

 (quadratic pattern)

Numerical

Graph

Table of Values

Describes same relationship

Take Action #1

Remember your goals:

- ★ - identify "growth" and "decay" factors (for relationships that are "exponential")
- ★ - represent growth and decay using different forms

Consider the following data set:

x	1	2	3	...
y	2	6	18	...

Handwritten annotations: Red arrows labeled '+1' above the x-values. Blue arrows labeled 'x3' below the y-values.

One of the later terms in the sequence of y-values is 4374.

Determine the previous two terms.

Solution: $\text{Growth Factor} = \frac{18}{6} = \frac{6}{2} = 3$

Let t_n be any term in the sequence, where n is a whole number.

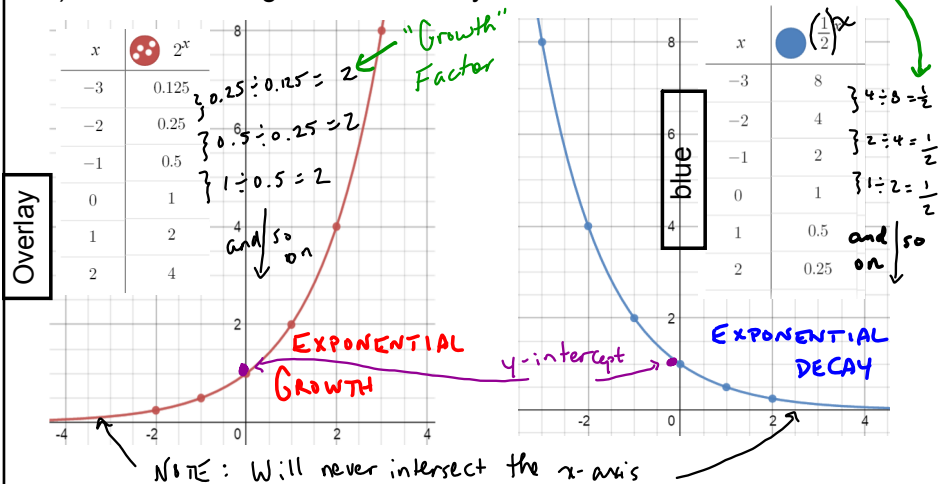
If $t_n = 4374$, then the previous two terms are given by t_{n-1} and t_{n-2} .

$$\begin{aligned} \text{So, } t_{n-1} &= \frac{t_n}{3} & ; \quad t_{n-2} &= \frac{t_{n-1}}{3} \\ &= \frac{4374}{3} & & = \frac{1458}{3} \\ &= 1458 & & = 486 \end{aligned}$$

\therefore The previous two terms to 4374 are 486 and 1458.

Take Action #2

- Which represents "growth"? "decay"?
- Determine the growth and decay factors.



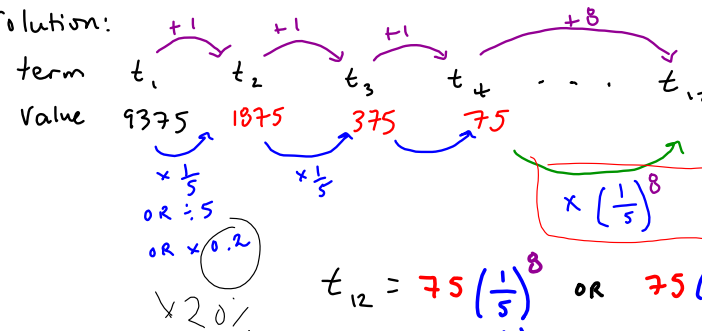
Take Action #3

A sequence has a decay factor of $\frac{1}{5}$. The first term in the sequence is 9375.

decay factor is a number that falls between 0 and 1.

- List the next three terms.
- Determine the value of the 12th term.

Solution:



BBC

$$\begin{aligned}
 t_{12} &= 75 \left(\frac{1}{5}\right)^8 \quad \text{OR} \quad 75 (0.2)^8 \\
 &= 75 \left(\frac{1}{5^8}\right) \\
 &= \frac{75}{5^8} \\
 &= 0.000192
 \end{aligned}$$

Calculator keystrokes: 0.2, y^x, 8, =, x, 75, =