

# Permutations -Lesson Objectives-

- 1) Independent vs. Dependent Events
- Learning Through Collaboration (Section 4.5, E.g., 3 to 5)
- Review homework to Section 4.5
- 2) Whole-class Exploration to Permutations
- 3) Define and Work with Factorial Notation (to facilitate calculations involving permutations)
- 4) Independent Practice (simplifying expressions involving factorials)

Lesson objectives

Teachers' notes



### 4.6-Permutations (Part 1)



During the last two sections of this chapter, we will be exploring two concepts that come to us from the field of counting possible outcomes--\_\_Combinatorics\_. In Part 2, of this lesson, we will connect the counting of possible outcomes to our study of probability.

The first, of two topics, is called <u>permutations</u>; the second, <u>combinations</u>.

Permutations are \_\_\_\_\_ordered \_\_\_ arrangements of a finite number of elements. For example, if we were interested in determining the total number of arrangements of n students or of a subset, r (i.e., r < n), of n students, we would be working with permutations.

Since permutations are ordered arrangements, two permutations that have exactly the same elements, but not in the same order, are treated as two, different arrangements or permutations.

Fade In

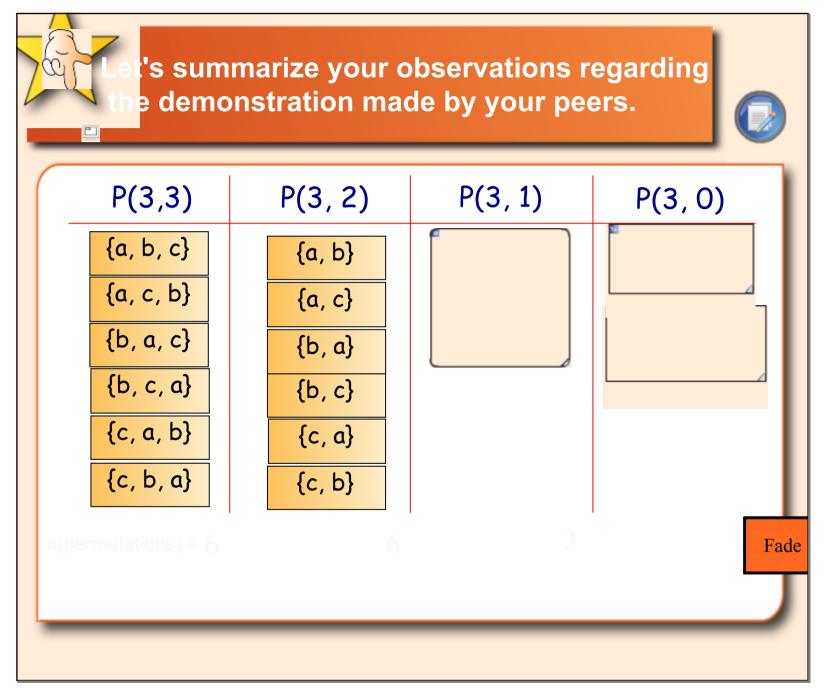


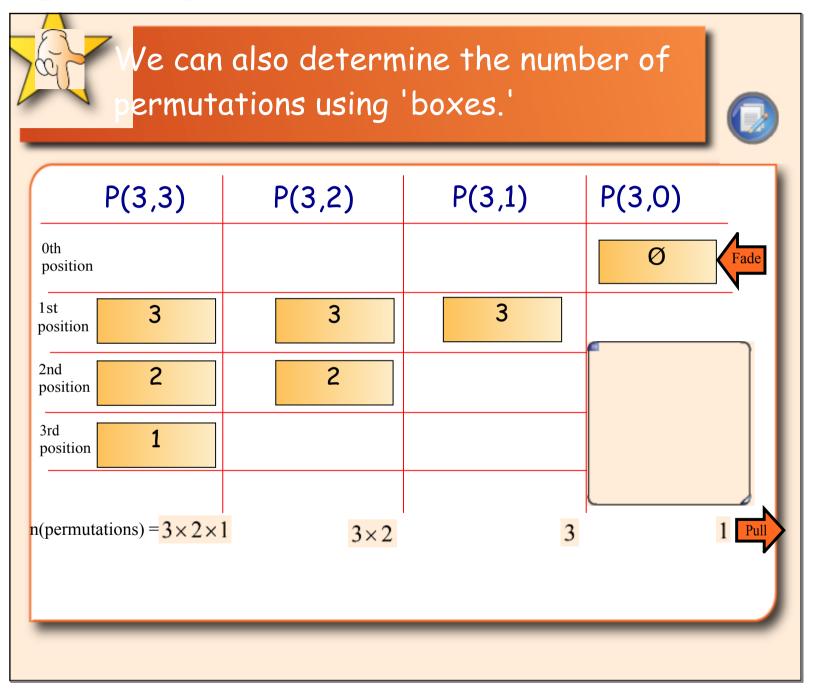
## Notations for Working With Permutations

Fade In



- Permutations are represented by the notations, P(n, n) and P(n, r).
- Other sources will use the notations, <sub>n</sub>P<sub>n</sub> and <sub>n</sub>P<sub>r</sub>.
- Writing <sub>n</sub>P<sub>r</sub> means "the number of permutations of *n* elements, selected *r* at a time" or "*r*-tuples of *n* items."





#### **Equation Notation for Permutations**

The following equation defines the number of *r*-arrangements of *n* items:



Let's apply this formula to calculate the number of *r*-arrangements determined from the demonstration.

#### Notes:

- The symbol, !, means "factorial."
- A factorial is the product of all positive integers less than or equal to *n*.
- E.g., n! = n(n-1)(n-2)(n-3)
- It follows that, for example,  $4! = 4 \times 3 \times 2 \times 1$



## Calculating permutations



P(3,3)

P(3,2)

P(3,1)

P(3,0)

3! (3-3)!

$$=\frac{3\times2\times1}{0!}$$

$$=\frac{-}{?}$$

=6

3! (3-2)!

$$=\frac{3\times2\times1}{1!}$$

$$=\frac{6}{1}$$

$$=6$$

3! (3-1)!

$$=\frac{3\times2\times1}{2!}$$

$$=\frac{6}{2\times1}$$

$$=3$$

3! (3-0)!

$$=\frac{3\times2\times1}{3!}$$

$$=\frac{6}{6}$$

Definition: 0! = 1

$$n! = n \times (n-1)!$$

Dividing both sides by *n* gives

$$\frac{n!}{n} = \frac{n \times (n-1)!}{n}$$

$$\frac{n!}{n} = (n-1)!$$

By setting n = 1

$$\frac{1!}{1} = (1-1)!$$

$$\frac{1}{1} = 0!$$

$$1 = 0!$$

#### E.g., 1. Simplifying Expressions Involving Factorials

Simplify and then evaluate each of the following:

$$\frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!}$$

Expand the largest factorial as far as the next largest factorial expression.

Cancel like factors (i.e., factorials).

$$=720$$

Evaluate.

$$\frac{6!}{4!2!} = \frac{6 \times 5 \times 4!}{4! \times 2 \times 1}$$

$$=\frac{6\times5}{2}$$

$$=15$$



**Erase to Reveal** 

#### E.g., 2. Solving an Equation Involving Factorials

Practice:#2, 3ace, 4eg, 7, 8ace, 9ac

• Scanned copy (see attached jpg), p1.

Solve for  $n \in W$ .

• Scanned copy (see attached jpg), p2

$$\frac{n!}{(n-2)!} = 20$$

$$\frac{n(n-1)(n-2)!}{(n-1)(n-2)!} = 20$$

Expand the largest factorial to that of the factorial in the denominator.

Cancel like factorials.

$$n(n-1)=20$$

$$n^2 - n = 20$$
 Expand.

$$n^2 - n - 20 = 0$$

— Collect all terms to one side such that the quadratic equation can be solved.

$$(n-5)(n+4) = 0$$
 Factor the quadratic.

$$n-5=0$$
 or  $n+4=0$ 

$$n = 5 \ or \ n = -4$$

$$\therefore$$
 For  $n \in W$ ,  $n = 5$ 

Note the restriction on *n* (i.e., *n* belongs to *W*-whole numbers only).

Factorial Notation\_p1.jpg

Factorial Notation\_p2.jpg