Agenda-Permutations (Part 1)

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1. Minds on...Learning Through Collaboration
2. Today's Learning Goals
3. Student Demonstration of Multiple Arrangements
4. Developing Working Rules for Arrangements
5. Applying Your Learning

Permutations
-Lesson Objectives-

1) Independent vs. Dependent Events
- Learning Through Collaboration (Section 4.5, E.g., 3 to 5)
- Review homework to Section 4.5

2) Whole-class Exploration to Permutations

3) Define and Work with Factorial Notation (to facilitate calculations involving permutations)

4) Independent Practice (simplifying expressions involving factorials)

Lesson objectives | Teachers' notes

4.6-Permutations (Part 1)

During the last two sections of this chapter, we will be exploring two concepts that come to us from the field of counting possible outcomes--_Combinatorics_. In Part 2, of this lesson, we will connect the counting of possible outcomes to our study of probability.

The first, of two topics, is called _permutations_; the second, _combinations_.

Permutations are _ordered_ arrangements of a finite number of elements. For example, if we were interested in determining the total number of arrangements of *n* students or of a subset, *r* (i.e., $r < n$), of *n* students, we would be working with permutations.

Since permutations are ordered arrangements, two permutations that have exactly the same elements, but not in the same _order_, are treated as two, different arrangements or permutations.

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Notations for Working With Permutations

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- Permutations are represented by the notations, P(n, n) and P(n, r).
- Other sources will use the notations, ${}_{n}P_{n}$ and ${}_{n}P_{r}$.
- Writing ${}_{n}P_{r}$ means "the number of permutations of *n* elements, selected *r* at a time" or "*r*-tuples of *n* items."

Let's summarize your observations regarding the demonstration made by your peers.

P(3,3)	P(3, 2)	P(3, 1)	P(3, 0)
{a, b, c}	{a, b}		
{a, c, b}	{a, c}		
{b, a, c}	{b, a}		
{b, c, a}	{b, c}		
{c, a, b}	{c, a}		
{c, b, a}	{c, b}		
n(permutations) = 6	6	3	1

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We can also determine the number of permutations using 'boxes.'

	P(3,3)	P(3,2)	P(3,1)	P(3,0)
0th position				Ø
1st position	3	3	3	
2nd position	2	2		
3rd position	1			
n(permutations) =	$3\times2\times1$	3×2	3	1

Fade

Pull

Equation Notation for Permutations

The following equation defines the number of *r*-arrangements of *n* items:

Let's apply this formula to calculate the number of *r*-arrangements determined from the demonstration.

Notes:

- The symbol, !, means "factorial."
- A factorial is the product of all positive integers less than or equal to *n*.
- E.g., $n! = n(n - 1)(n - 2)(n - 3)$
- It follows that, for example, 4! = 4 x 3 x 2 x 1

Pull

Calculating permutations

P(3,3)	P(3,2)	P(3,1)	P(3,0)
$\frac{3!}{(3-3)!}$ $=\frac{3\times2\times1}{0!}$ $=\frac{6}{?}$ $=6$	$\frac{3!}{(3-2)!}$ $=\frac{3\times2\times1}{1!}$ $=\frac{6}{1}$ $=6$	$\frac{3!}{(3-1)!}$ $=\frac{3\times2\times1}{2!}$ $=\frac{6}{2\times1}$ $=3$	$\frac{3!}{(3-0)!}$ $=\frac{3\times2\times1}{3!}$ $=\frac{6}{6}$ $=1$

Definition: 0! = 1

$$n! = n \times (n-1)!$$

Dividing both sides by *n* gives

$$\frac{n!}{n} = \frac{n \times (n-1)!}{n}$$

$$\frac{n!}{n} = (n-1)!$$

By setting *n* = 1

$$\frac{1!}{1} = (1-1)!$$

$$\frac{1}{1} = 0!$$

$$\boxed{1 = 0!}$$

E.g., 1. Simplifying Expressions Involving Factorials

Simplify and then evaluate each of the following:

$$\frac{10!}{7!} = \frac{10\times9\times8\times7!}{7!}$$ ← Expand the largest factorial as far as the next largest factorial expression.

$$= 10\times9\times8$$ ← Cancel like factors (i.e., factorials).

$$= 720$$ ← Evaluate.

$$\frac{6!}{4!2!} = \frac{6\times5\times4!}{4!\times2\times1}$$

$$= \frac{6\times5}{2}$$

$$= 15$$

Erase to Reveal

E.g., 2. Solving an Equation Involving Factorials

Practice:#2, 3ace, 4eg, 7, 8ace, 9ac
- Scanned copy (see attached jpg), p1.
- Scanned copy (see attached jpg), p2

Solve for $n \in$ W.

$$\frac{n!}{(n-2)!} = 20$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 20$$

← Expand the largest factorial to that of the factorial in the denominator.

Cancel like factorials.

$$n(n-1) = 20$$

$$n^2 - n = 20$$

← Expand.

$$n^2 - n - 20 = 0$$

← Collect all terms to one side such that the quadratic equation can be solved.

$$(n-5)(n+4) = 0$$

← Factor the quadratic.

$$n-5=0 \ or \ n+4=0$$

$$n=5 \ or \ n=-4$$

$$\therefore For\ n \in W, n = 5$$

← Note the restriction on *n* (i.e., *n* belongs to *W*-- whole numbers only).

Attachments

Factorial Notation_p1.jpg

Factorial Notation_p2.jpg