

# Permutations

## -Lesson Objectives-

- to develop an understanding of the concept of a permutation
- to build skills simplifying expressions involving factorial notation
- to see the numerical representation of a permutation as an expression involving factorials

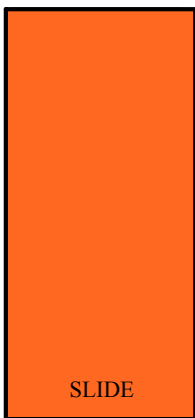
Lesson objectives

Teachers' notes

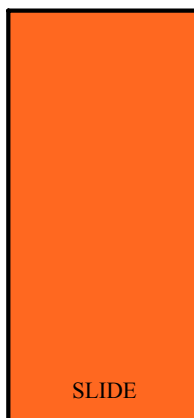


Let's summarize your observations regarding the demonstration made by your peers.

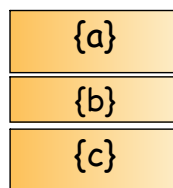
$P(3,3)$



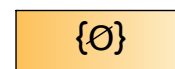
$P(3,2)$



$P(3,1)$



$P(3,0)$



=> 1 way to select no items

(permutations) = 6

6

3

Fade



We can also determine the number of permutations using 'boxes.'

	$P(3,3)$	$P(3,2)$	$P(3,1)$	$P(3,0)$
0th position				0
1st position	3	3	3	
2nd position	2	2		
3rd position	1			
				$P(n, r)$ number of items $\swarrow$ $n$ positions to be filled by elements $\searrow$ $r$
n(permutations) =	$3 \times 2 \times 1$	$3 \times 2$	3	1



## Permutations (Part 1)

During the last two sections of this unit, we will be exploring two concepts that come to us from the field of counting possible outcomes--**Combinatorics**. In Part 2, of this lesson, we will connect the counting of possible outcomes to our study of probability.

The first, of two topics, is called **permutations**; the second, **combinations**.

Permutations are **ordered** arrangements of a finite number of elements. For example, if we were interested in determining the total number of arrangements of  $n$  students or of a subset,  $r$  (i.e.,  $r < n$ ), of  $n$  students, we would be working with permutations.

Since permutations are ordered arrangements, two permutations that have exactly the same elements, but not in the same **order**, are treated as two, different arrangements or permutations.





## Notations for Working With Permutations

- Permutations are represented by the notations,  $P(n, n)$  and  $P(n, r)$ .
- Other sources will use the notations,  ${}_n P_n$  and  ${}_n P_r$ .
- Writing  ${}_n P_r$  means "the number of permutations of  $n$  elements, selected  $r$  at a time" or " $r$ -tuples of  $n$  items."



## Calculating permutations

$P(3,3)$

$$\begin{aligned} & \frac{3!}{(3-3)!} \\ &= \frac{3 \times 2 \times 1}{0!} \\ &= \frac{6}{1} \\ &= 6 \end{aligned}$$

$P(3,2)$

$$\begin{aligned} & \frac{3!}{(3-2)!} \\ &= \frac{3 \times 2 \times 1}{1!} \\ &= \frac{6}{1} \\ &= 6 \end{aligned}$$

$P(3,1)$

$$\begin{aligned} & \frac{3!}{(3-1)!} \\ &= \frac{3 \times 2 \times 1}{2!} \\ &= \frac{6}{2 \times 1} \\ &= 3 \end{aligned}$$

$P(3,0)$

$$\begin{aligned} & \frac{3!}{(3-0)!} \\ &= \frac{3 \times 2 \times 1}{3!} \\ &= \frac{6}{6} \\ &= 1 \end{aligned}$$

Definition:  $0! = 1$

$$n! = n \times (n-1)!$$

Dividing both sides by  $n$  gives

$$\frac{n!}{n} = \frac{n \times (n-1)!}{n}$$

$$\frac{n!}{n} = (n-1)!$$

By setting  $n = 1$

$$\frac{1!}{1} = (1-1)!$$

$$\frac{1}{1} = 0!$$

$$1 = 0!$$

### Equation Notation for Permutations

The following equation defines the number of arrangements of  $n$  items:

$$P(n, r) = \frac{n!}{(n-r)!}$$

Let's apply this formula to calculate the number of arrangements determined from the demonstration.

Pull

### E.g., 1. Simplifying Expressions Involving Factorials

Simplify and then evaluate each of the following:

$$\frac{10!}{7!} =$$



Expand the largest factorial as far as the next largest factorial expression.

=



Cancel like factors (i.e., factorials).

=



Evaluate.

---

$$\frac{6!}{4!2!} =$$

=

=



Erase to Reveal

### E.g., 2. Solving an Equation Involving Factorials

Solve for  $n \in \mathbb{W}$ .

$$\frac{n!}{(n-2)!} = 20$$



## Attachments

---

Factorial Notation\_p1.jpg

Factorial Notation\_p2.jpg