Permutations
-Lesson Objectives-

-to develop an understanding of the concept of a permutation

-to build skills simplifying expressions involving factorial notation

-to see the numerical representation of a permutation as an expression involving factorias

Lesson objectives

Teachers' notes

Let's summarize your observations regarding the demonstration made by your peers.

P(3,3)	P(3, 2)	P(3, 1)	P(3, 0)
SLIDE	SLIDE	{a} {b} {c}	{Ø} => 1 way to select no items
n(permutations) =6	6	3	1

Fade

We can also determine the number of permutations using 'boxes.'

	$P(3,3)$	$P(3,2)$	$P(3,1)$	$P(3,0)$
0th position				$\emptyset$
1st position	3	3	3	
2nd position	2	2		
3rd position	1			
n(permutations) =	$3 \times 2 \times 1$	3×2	3	1

Fade

$P(n, r)$

number of items

positions to be filled by elements

Pull

Permutations (Part 1)

During the last two sections of this unit, we will be exploring two concepts that come to us from the field of counting possible outcomes--_Combinatorics_. In Part 2, of this lesson, we will connect the counting of possible outcomes to our study of probability.

The first, of two topics, is called _permutations_; the second, _combinations_.

Permutations are _ordered_ arrangements of a finite number of elements. For example, if we were interested in determining the total number of arrangements of *n* students or of a subset, *r* (i.e., r < n), of *n* students, we would be working with permutations.

Since permutations are ordered arrangements, two permutations that have exactly the same elements, but not in the same _order_, are treated as two, different arrangements or permutations.

Notations for Working With Permutations

- Permutations are represented by the notations, P(n, n) and P(n, r).
- Other sources will use the notations, $_nP_n$ and $_nP_r$.
- Writing $_nP_r$ means "the number of permutations of *n* elements, selected *r* at a time" or "*r*-tuples of *n* items."

Calculating permutations

P(3,3)	P(3,2)	P(3,1)	P(3,0)
$\frac{3!}{(3-3)!}$ $=\frac{3\times2\times1}{0!}$ $=\frac{6}{?}$ $=6$	$\frac{3!}{(3-2)!}$ $=\frac{3\times2\times1}{1!}$ $=\frac{6}{1}$ $=6$	$\frac{3!}{(3-1)!}$ $=\frac{3\times2\times1}{2!}$ $=\frac{6}{2\times1}$ $=3$	$\frac{3!}{(3-0)!}$ $=\frac{3\times2\times1}{3!}$ $=\frac{6}{6}$ $=1$

$$\boxed{\text{Definition: } 0! = 1}$$

$$n! = n \times (n-1)!$$

Dividing both sides by *n* gives

$$\frac{n!}{n} = \frac{n \times (n-1)!}{n}$$

$$\frac{n!}{n} = (n-1)!$$

By setting *n* = 1

$$\frac{1!}{1} = (1-1)!$$

$$\frac{1}{1} = 0!$$

$$\boxed{1 = 0!}$$

Equation Notation for Permutations

The following equation defines the number of-arrangements of *n* items:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Let's apply this formula to calculate the number of-arrangements determined from the demonstration.

Pull

E.g., 1. Simplifying Expressions Involving Factorials

Simplify and then evaluate each of the following:

$\frac{10!}{7!} =$ ← Expand the largest factorial as far as the next largest factorial expression.

$=$ ← Cancel like factors (i.e., factorials).

$=$ ← Evaluate.

$\frac{6!}{4!2!} =$

$=$

$=$

Erase to Reveal

E.g., 2. Solving an Equation Involving Factorials

Solve for $n \in$ W.

$$\frac{n!}{(n-2)!} = 20$$

P

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Attachments

Factorial Notation_p1.jpg

Factorial Notation_p2.jpg