Permutations_Solving Problems with...

Example 1: Counting Arrangements

Consider the following example:

In how many ways can the starting order, for the NG track team, be posted in an 8-member relay if ...

a) all 8 members will take part in the race?

b) only 4 members, chosen from the 8, will take part?

a) 8 7 6 5 4 3 2 1

n(arrangements) = 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 8!
= 40 320

$_8P_8 = P(8,8) = 40\,320$

Permutations: number of finite arrangements of 8 items, selecting all 8 items.

b) 8 7 6 5

n(arrangements)
$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$.

$_8P_4 = \frac{8!}{4!}$
$= 1680$

number of arrangements (i.e., permutations) of 8 items, selecting any 4 items.

$_nP_r = \frac{n!}{(n-r)!}$ using formula from slide 2.

$_8P_4 = \frac{8!}{(8-4)!}$
$= \frac{8!}{4!}$
$= 1680$

Formulas for Permutations

The number of permutations of n objects, taken r at a time, and without repetition is given by:

$$P(n,r) = \frac{n!}{(n-r)!}, \quad 0 \le r \le n$$

Also...

$$P(n,n) = \frac{n!}{(n-n)!}$$

$$P(n,n) = \frac{n!}{0!}$$

$$P(n,n) = \frac{n!}{1} \quad \text{where } 0! = 1$$

$$P(n,n) = n!$$

Example 2: Using Permutations to Solve Probability-related Problems

Re-consider the previous example:

In how many ways can the starting order, for the NG track team, be posted in an 8-member relay if ...

... only 4 members, chosen from the 8, will take part?

NOW...

Let's say that you'd like to know the probability that you and 3 of your friends (also on the track team) will form the starting order.

Determine this probability. Let A be the event that you and 3 friends are chosen.

$$P(A) = \frac{n(A)}{n(S)} = \frac{4 \times 3 \times 2 \times 1}{{}_8P_4} = \frac{4!}{{}_8P_4} = \frac{24}{1680} = \frac{1}{70}$$

Example 3: Permutations with Identical Elements

List the permutations of the letters in the word, ROOM.

R O_1 O_2 M

SEE pictures posted to the course webpage.

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Permutations with Identical Elements

When all objects are not distinct

The number of permutations of *n* objects, for which there are *a* identical elements, *b* identical elements, and so on..., the number of permutations is given by the following:

$$\frac{n!}{a!b!c!\ldots} = \frac{{}_nP_n}{a!b!c!\ldots}$$

E.g. 4., Determine the number of possible arrangements using the letters of the word, MINIMUM.

$$n(\text{arrangements}) = \frac{{}_7P_7}{2!\,3!}$$

$$= \frac{7\times6\times5\times4\times3!}{2!\,3!}$$

NOTE: $\frac{3!}{3!} = \frac{1}{1}$

$$= \frac{7\times\cancel{6}^{3}\times5\times4}{\cancel{2}\times1}$$

NOTE: $\frac{6}{2} = \frac{3}{1}$

$$= 420$$

Practice Problem Set

p255 #4, 6, 7, 11, 12a, 16, 17