

Chapter 4 Practice Problems

Multiple Choice (Knowledge & Understanding)

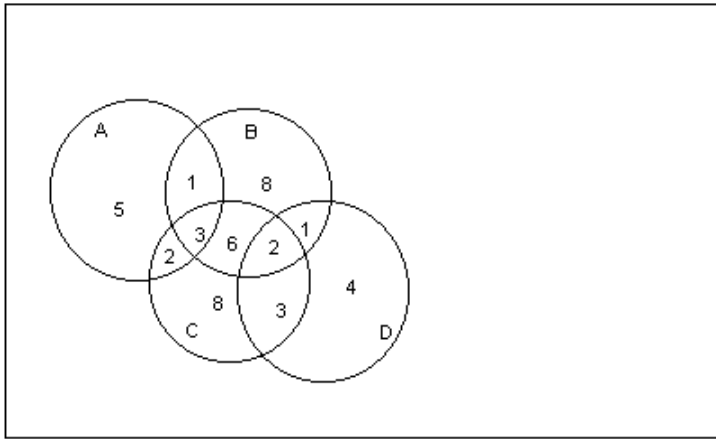
Identify the choice that best completes the statement or answers the question.

- _____ 1. An experiment that models an actual event is a(n)
a. trial
b. test
c. simulation
d. experiment
- _____ 2. The chances of a family having a boy is simulated by tossing a coin ten times. Heads is used to simulate having a girl and tails is used to simulate having a boy. Out of ten trials, two heads and eight tails turn up. This experimental probability of $\frac{8}{10}$ is
a. close to the theoretical probability
b. equal to the theoretical probability
c. not close to the theoretical probability
d. none of the above
- _____ 3. Two friends play a game. They roll a die. If the roll is 1 or 2, the first friend gives the second friend \$3.00. If the roll is a 6, the second friend gives the first friend \$5.00. The expected amount of money the first friend receives if the die is rolled 12 times is
a. \$5.00
b. \$10.00
c. \$3.00
d. \$6.00
- _____ 4. A spinner is divided into twelve equal sectors, numbered 1 through 12. An event space is defined as spinning a number divisible by 3. The value of $P(A')$ is
a. $\frac{3}{4}$
b. $\frac{1}{3}$
c. $\frac{1}{4}$
d. $\frac{2}{3}$
- _____ 5. Each of the letters of the word PROBABILITY are placed in a hat. If one letter is drawn randomly, what is the probability that a vowel is pulled out?
a. $\frac{3}{11}$
b. $\frac{8}{11}$
c. $\frac{4}{11}$
d. $\frac{7}{11}$
- _____ 6. Which of the following statements is true?
a. $n(A) \leq n(S)$
b. $P(A') = 1 + P(A)$
c. $n(A) = 1 - n(A')$
d. none of the above
- _____ 7. Let set A be the set of even numbers between 1 and 20. Let B be the set of all numbers divisible by three between 1 and 20. Which statement is true?
a. $n(A \cap B) = 3$
b. $n(A \cup B) = 6$
c. $n(A \cap B) = 6$
d. $n(A \cup B) = 3$
- _____ 8. Which of the following is the additive principle for probabilities?
a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
b. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
c. $P(A \cup B) = P(A) + P(B) + P(A \cap B)$
d. $P(A \cap B) = P(A) + P(B) + P(A \cup B)$
- _____ 9. The probability that John will be accepted into the business program at Probability College is 0.6. The probability that he will be accepted into the science program is 0.2. The probability that he will be accepted into both programs is 0.1. What is the probability that he will be accepted into at least one of the programs?
a. 0.7
b. 0.9
c. 0.12
d. none of the above

- _____ 19. The expression $\binom{7}{2}$ is equivalent to
- a. 42
b. $\frac{7!}{2!}$
c. 21
d. none of the above
- _____ 20. Determine which of the following problems must be determined using a combination.
- Determine the probability of drawing an ace and a four from a deck of cards in that order.
 - Determine the probability that Rhys, Lorne, Cheryl, and Carl are chosen when a group of six is chosen from nineteen people.
 - Determine the probability of rolling doubles when a pair of dice are rolled.
 - Determine the number of ways that a captain and co-captain can be chosen from a football team of 35 members.

Short Answer (Knowledge & Understanding)

- A CD is defective 3% of the time. A simulation is done by a spreadsheet which produces random numbers from 1 to 100. What are the possible numbers that represent a non-defective CD?
- A die is rolled and a coin is flipped at the same time. How many possible events can be simulated using this method?
- A bag contains 4 red marbles, 3 blue marbles, 6 black marbles, and 2 yellow marbles. If a marble is drawn at random, determine the probability of not drawing a red marble.
- Two dice are rolled. The probability of rolling doubles is to be determined. State $n(A')$.



- According to the following Venn diagram, what is the value of $n(A \cup B)$?
- Two dice are rolled. Determine the probability of rolling pairs or a sum of four.
- Out of a group of people surveyed, 70 say they read the newspaper and 83 say they watch television. If 25 say they do both, draw a Venn diagram to represent this scenario. A is the set of people who read the newspaper and B is the set of people who watch T.V. Assume that every person surveyed is involved in at least one of these activities.
- A survey indicated that 60% of the people watch television. Of those people, 70% watch more than one hour per day and the rest watch more than one hour. Determine the probability that a person, chosen at random, watches more than one hour of television per day.
- Determine the probability of drawing a red card and then the ace of spades from a regular deck of cards, given that the red card is not returned to the deck.

30. If $P(A) = \frac{4}{7}$, $P(B) = \frac{6}{11}$, and $P(A \cap B) = \frac{2}{13}$, determine $P(B|A)$.
31. A box contains one each of a math, english, history, and science textbook. One textbook is drawn randomly and then replaced. Then a second textbook is drawn out. Draw a tree diagram to represent the possible outcomes.
32. Express $40 \times 39 \times 38$ using factorials.
33. State the number of ways that the 9 members of the debating club can be lined up for a picture if Frasier must be on the far left and Samantha and Charlotte must be together.
34. Solve for n if $\frac{n!}{(n-1)!} = 42$.
35. Write ${}_{13}C_5$ in terms of factorials.
36. A committee of 3 students is chosen from 6 music students and 5 drama students. Determine the probability that exactly 2 are drama students.

Application Problems

37. A traffic light is red for 35 s, green for 60 s, and amber for 5 s. Design a simulation using a spinner to determine the probability that, at the moment a car arrives at the intersection, the light will be green.
38. A student is writing a test with 4 true/false questions. The student wants to know the theoretical probability of passing the test by guessing. This means that 2 or more of the 4 questions are answered correctly by guessing. Determine this probability and show all your work.
39. A game show has three doors labelled A, B, and C behind which there may or may not be a prize. A group of contestants are asked behind which doors they think there is a prize. The results are: 18 people choose A, 19 people choose B, 19 people choose C, 4 people choose A and B, 7 people choose A and C, 10 people choose B and C, and 3 people choose all three. Determine the number of people who only choose A.
40. A group of 200 candidates apply for a job. Only 10 will be given interviews and only 3 of those candidates will be given second interviews. If the selection process is completely random, what is the probability of being given a second interview?
41. If a family has five children, determine the probability that two of the children are girls.
42. The first line of a local all-star team is to be composed of 4 strikers, 3 midfielders, and 3 defencemen. There are 20 strikers, 17 midfielders, and 15 defencemen trying out. Determine the probability that Steven, a striker, and Paul, a defenceman, both make the starting lineup.

Answer Section

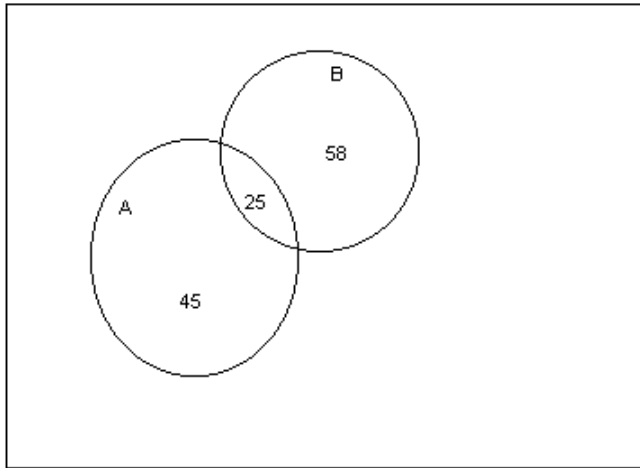
MULTIPLE CHOICE

1. ANS: C
2. ANS: A
3. ANS: B
4. ANS: B
5. ANS: C
6. ANS: A
7. ANS: D
8. ANS: A
9. ANS: A
10. ANS: D
11. ANS: B
12. ANS: B
13. ANS: C
14. ANS: A
15. ANS: C
16. ANS: D
17. ANS: C
18. ANS: B
19. ANS: C
20. ANS: B

SHORT ANSWER

21. ANS:
The possible numbers that represent a non-defective CD are 4 to 100.
22. ANS:
The number of possible events that can be simulated using this method is 12.
23. ANS:
The probability of not drawing a red marble is $\frac{11}{15}$.
24. ANS:
The value of $n(A')$ is 30.
25. ANS:
The value of $n(A \cup B)$ is 28.
26. ANS:
The probability of rolling pairs or a sum of four is $\frac{2}{9}$.

27. ANS:



28. ANS:

The probability that a person watches more than one hour of television per day is 0.42.

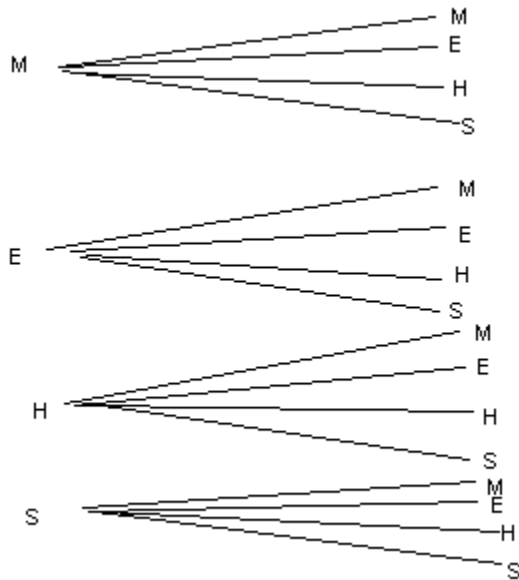
29. ANS:

The probability of drawing a red card and then the ace of spades is $\frac{1}{102}$.

30. ANS:

$$P(B|A) = \frac{7}{26}.$$

31. ANS:



32. ANS:

Using factorials, $40 \times 39 \times 38$ is $\frac{40!}{37!}$.

33. ANS:
The number of ways is 10 080.

34. ANS:
 $n = 42$

35. ANS:
In terms of factorials, ${}_{13}C_5$ is $\frac{13!}{8!5!}$.

36. ANS:
The probability is $\frac{4}{11}$.

PROBLEM

37. ANS:
Divide the spinner into 3 sections. The red sector will have an angle of $\frac{35}{100} \times 360^\circ = 126^\circ$. The green sector will have an angle of $\frac{60}{100} \times 360^\circ = 216^\circ$. The amber sector will have an angle of $\frac{5}{100} \times 360^\circ = 18^\circ$. Each trial will consist of one spin of the spinner. The arrow landing in the red sector will simulate arriving at a red light, and likewise for the other two colours. Perform many trials and calculate the experimental probability.

38. ANS:
Let A be the event of answering two or more questions correctly.
 $A = \{(\text{question 1 correct, question 2 correct}), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4), (1, 2, 3, 4)\}$
Therefore, $n(A) = 11$.

$$n(S) = 2 \text{ choices} \times 2 \text{ choices} \times 2 \text{ choices} \times 2 \text{ choices}$$

$$= 16$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{11}{16}$$

39. ANS:

Let A , B , and C be the sets of people choosing doors A, B, and C.

$$\begin{aligned}n(\text{only } A) &= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) \\ &= 18 - 4 - 7 + 3 \\ &= 10\end{aligned}$$

40. ANS:

Let A be the event of being given a first interview and B be the event of being given a second interview. The probability of being given a second interview is:

$$P(A \cap B) = P(A) \times P(B|A)$$

$$\begin{aligned}&= \frac{10}{200} \times \frac{3}{10} \\ &= \frac{30}{2000} \\ &= \frac{3}{200}\end{aligned}$$

41. ANS:

The probability of having a girl or a boy is $\frac{1}{2}$ and the number of ways of having two girls out of five children is 10 which can be seen with a tree diagram or using another counting method.

$$\begin{aligned}\therefore P(2 \text{ are girls}) &= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) \times 10 \\ &= \frac{5}{16}\end{aligned}$$

42. ANS:

$$\begin{aligned}P(\text{both players make the lineup}) &= \frac{\binom{19}{3} \binom{17}{3} \binom{14}{2}}{\binom{20}{4} \binom{17}{3} \binom{15}{3}} \\ &= \frac{59\,961\,720}{1\,499\,043\,000} \\ &= \frac{1}{25}\end{aligned}$$