

## Chapter 5 Practice Problems

### Multiple Choice (Knowledge & Understanding)

Identify the choice that best completes the statement or answers the question.

- \_\_\_\_\_ 1. The probability of drawing a red card from a deck of 52 playing cards is 0.5. Which of the following statements is not necessarily true?
- The expected value for the number of red cards in a 7 card hand will be 3.5.
  - Over many repeated drawings you would expect the ratio of red cards drawn to total number of drawings to be close to 1:2.
  - The expected value for the number of black cards in an 8 card hand will be 4.
  - If you successively draw and replace a card 100 times, you will see 50 red cards.
- \_\_\_\_\_ 2. Which of the following is an example of a discrete random variable?
- the time needed by a student to complete a physics lab
  - the number of red cars observed in a student parking lot
  - the mass of a green pepper selected at random from a bin
  - the cost of a U.S. dollar in Canadian currency on a given day
- \_\_\_\_\_ 3. The following table shows the probability distribution for the possible sums that result from rolling two 6-sided dice.

$X$	$P(X)$	$X$	$P(X)$
1	0	7	$\frac{6}{36}$
2	$\frac{1}{36}$	8	$\frac{5}{36}$
3	$\frac{2}{36}$	9	$\frac{4}{36}$
4	$\frac{3}{36}$	10	$\frac{3}{36}$
5	$\frac{4}{36}$	11	$\frac{2}{36}$
6	$\frac{5}{36}$	12	$\frac{1}{36}$

What is the probability that the sum rolled is even and less than 9?

- $\frac{7}{18}$
  - $\frac{5}{36}$
  - $\frac{1}{9}$
  - $\frac{1}{2}$
- \_\_\_\_\_ 4. The single expression of the form  $\binom{n}{r}$ , which is equivalent to  $\binom{11}{3} + \binom{11}{4}$ ?
- $\binom{11}{5}$
  - $\binom{12}{4}$

b.  $\binom{12}{3}$

d.  $\binom{121}{12}$

- \_\_\_ 5. Which of the following is not a property of a Binomial Experiment?
- All trials are identical.
  - Each trial has only two possible outcomes.
  - The probability of success may change from trial to trial.
  - The purpose of the experiment is to determine the number of successes that occurs during the  $n$  trials.
- \_\_\_ 6. In the expression  $\binom{8}{3}(0.2)^3(0.8)^5$ , which value represents the number of trials?
- 2
  - 3
  - 5
  - 8
- \_\_\_ 7. In the expression  $\binom{10}{3}(0.5)^3(0.5)^7$ , which value represents the number of successes?
- 3
  - 10
  - 5
  - 7
- \_\_\_ 8. A young couple plans to have a family with four children. Assuming that the behaviour of their first child does not cause them to alter their plans, what is the expected number of girls for their family?
- 2.5
  - 2.25
  - 2
  - 1.5
- \_\_\_ 9. The probability of success for a binomial experiment is greater than 0.5. Which is the most accurate description of the graph of its probability distribution?
- symmetrical
  - highest point is left of centre
  - highest point is right of centre
  - all bars have equal height

**Short Answer (Knowledge & Understanding)**

Answer the following on lined paper. Practice effective communication.

- What is the expected value for the number of red cards found in a hand of 5 cards dealt from a deck of 52 cards?
- Calculate the expected value for the given probability distribution.

$X$	$P(X)$
6	0.15
8	0.35
10	0.5

- From the top right hand corner of a 5 by 5 square checkerboard, how many paths will finish in the middle square in the bottom row?
- A fair die has four faces numbered one to four. What is the probability of rolling a two exactly three times in ten rolls of the die?

### Application Problems

Answer the following on lined paper. Practice effective communication.

14. The information in the table below was extracted from the 1996 Canadian census.

<b>Number of Persons Living in Household</b>	<b>Relative Frequency</b>
1	0.36
2	0.28
3	0.14
4	0.15
5	0.05
>5	0.02

(use 6.3 for average)

- a) Based on this information, predict the expected population of a city that had a total of 9 000 households.  
b) Explain why using this information to predict the population of an individual street in the city could be unreliable.
15. The eighteenth hole at a public golf course is a par 4. This means that a good golfer should be able to play the hole in 4 strokes. The table below summarizes the scores of a sample of 200 golfers who recently played the hole.

<b>No. of Strokes</b>	1	2	3	4	5	6	7	8	9	10
<b>Frequency</b>	0	3	24	45	51	38	24	9	5	1

- a) What is the expected value for the number of strokes a golfer will take on this hole?  
b) Is it reasonable that the expected value for the hole should differ from the par value?
16. A student writes a five question multiple-choice quiz. Each question has four possible responses. The student guesses at random for each question. Calculate the probability for each possible score on the test from 0 to 5.
17. A small math class consists of 16 students. What is the probability that the difference in the number of male and female students in the class is greater than 4?

## Answer Section

### MULTIPLE CHOICE

1. ANS: D
2. ANS: B
3. ANS: A
4. ANS: C
5. ANS: C
6. ANS: D
7. ANS: A
8. ANS: C
9. ANS: C

### SHORT ANSWER

10. ANS:  
2.5

11. ANS:  
8.7

12. ANS:  
Three.

				X
			1	
		1		1
	1		2	
		3		

13. ANS:  
0.2503

### PROBLEM

14. ANS:
- a) The simplest method is to use the table to get the average number of persons per household and then multiply by the number of households.  
Average per household =  $(1)(0.36) + (2)(0.28) + (3)(0.14) + (4)(0.15) + (5)(0.05) + (6.3)(0.02) = 2.316$   
For 9000 households, the population would be 20 844.
  - b) For an individual street, the housing mix is unlikely to be representative of the overall national average that is reflected in the table. For example, a typical residential street would be highly unlikely to have 36% of the houses with a single occupant. Similarly, a street with many apartment buildings would be likely to have a smaller frequency of larger family units than the national average.

15. ANS:

$$\begin{aligned} \text{a) } E(X) &= 1(0) + 2\left(\frac{3}{200}\right) + 3\left(\frac{24}{200}\right) + 4\left(\frac{45}{200}\right) + 5\left(\frac{51}{200}\right) + 6\left(\frac{38}{200}\right) + 7\left(\frac{24}{200}\right) + 8\left(\frac{9}{200}\right) + 9\left(\frac{5}{200}\right) + \\ &10\left(\frac{1}{200}\right) = 5.18 \end{aligned}$$

The expected number of strokes for the hole is 5.18.

b) Most golfers are not good golfers. Unless the pro tour was visiting, we would not expect the average score for the hole to approach par.

16. ANS:

$$P(X=0) = \binom{5}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 = 0.2373$$

$$P(X=1) = \binom{5}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^4 = 0.3955$$

$$P(X=2) = \binom{5}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 = 0.2637$$

$$P(X=3) = \binom{5}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = 0.0879$$

$$P(X=4) = \binom{5}{4} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^1 = 0.0146$$

$$P(X=5) = \binom{5}{5} \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0 = 0.0010$$

17. ANS:

There are five cases where the gender split is 4 or less. These are 8M/8F, 9M/7F, 9F/7M, 10M/6F, and 10F/6M. Subtract the sum of these probabilities from 1 to obtain the requested probability. Define  $X$  as the random variable representing the number of males in the class.

$$P(X=8) = \binom{16}{8} (0.5)^8 (0.5)^8 = 0.1964$$

$$P(X=7) = P(X=9) = \binom{16}{9} (0.5)^9 (0.5)^7 = 0.1746$$

$$P(X=6) = P(X=10) = \binom{16}{10} (0.5)^{10} (0.5)^6 = 0.1222$$

$$P(X = 6, 7, 8, 9, 10) = 0.1964 + 2(0.1746) + 2(0.1222) = 0.79$$

This represents the probability that the gender split is 4 or less. The requested probability is 0.21.