

Solution Let the random variable X be the number of tagged fish caught. The probability distribution is hypergeometric where

$$\begin{aligned} a &= \text{the number of tagged fish in the lake} \\ &= 1000 \end{aligned}$$

$$b = \text{the number of untagged fish}$$

$$\begin{aligned} n &= \text{the number of fish caught} \\ &= 500 \end{aligned}$$

The 60 fish that were caught provide an estimate of $E(X)$.

To estimate the number of fish in the lake we solve for the unknown variable b in the equation for the expectation

$$E(X) = \frac{na}{a+b}$$

Substituting the known information we have,

$$60 = \frac{500 \times 1000}{1000 + b}$$

Solving for b , we obtain

$$60(1000) + 60b = 500\,000$$

$$60b = 500\,000 - 60\,000$$

$$60b = 440\,000$$

$$b = 7\,000$$

An estimate of the total number of fish in the lake is

$$\begin{aligned} a + b &= 1000 + 7000 \\ &= 8000 \end{aligned}$$



EXERCISE 6.5

HYPERGEOMETRIC DISTRIBUTIONS

- A 1. Identify the probability distribution in which the following random variables are distributed.
- Let the random variable X be the number of red cards drawn in the drawing of cards from a deck.
 - Let the random variable X be the number of times a coin is tossed before a head is tossed.
 - Let the random variable X be the number of red Smarties randomly drawn from a handful of Smarties.
 - Let the random variable X be the number of people that are born on a Monday in a class of 40 students.
2. A fish bowl contains ten guppies and eight goldfish. Four of the fish are removed randomly. Identify the values of the variables a , b , n , and x , if you were to find the probability that two of the fish removed were guppies.

Source: ?

Stewart, J., et al. (1998). *Finite Mathematics*. McGraw-Hill
 Ryerson: Toronto. pp. 252-254, 529.

- B 3. (a) Tabulate and graph the hypergeometric distributions for the following cases.
- $a = 5, b = 5, n = 4$
 - $a = 5, b = 10, n = 4$
 - $a = 10, b = 5, n = 4$
- (b) For each of the distributions of part (a), verify that $\sum_x p(x) = 1$ and also find the expectation for each case. Mark the expectation on your graph of the distribution.
4. A marble bag contains ten "steelies" and 15 "cat's eyes." If Lawrie reaches into the bag and withdraws five marbles, what is the probability that she will get:
- exactly one steelie?
 - at least two steelies?
 - no cat's eyes?
5. In a club with 20 members, five have positions of authority. If a random poll of ten of the club members is taken, what is the probability that at least two of those polled have positions of authority?
6. A poker hand of five cards is dealt.
- What is the probability that the hand does not contain any face cards?
 - Answer the same question for a bridge hand of 13 cards.
7. Nick organized a school fund-raising draw to raise money for the purchase of a rowing shell. There were 500 tickets sold at \$5.00 each. Two hundred of the tickets were sold internally to members of the student body and the remainder were sold in the community at large. If three prizes were drawn, what is the probability that
- none of the prizes were won by students?
 - all of the prizes were won by students?
8. At an office party names were drawn out of a hat to pick teams for charades. There were ten females and six males at the party. What is the probability that the first team drawn (four members to a team) was all female?
9. In a special education class of 12 students, four have advanced aptitudes in Mathematics. If a ministry official randomly tests three students, what is the probability that all three of the students tested are gifted in Mathematics?
10. A box contains 80 good and ten defective screws. If ten screws are used, what is the probability that none are defective?
11. What is the probability of having a four-three-three-three distribution of cards (spades-hearts-diamonds-clubs, respectively) in a bridge hand?
12. What is the probability of being dealt a straight (not necessarily all of the same suit) from a ten to the ace in a poker hand?

13. A bag of jelly beans contains ten red, 12 yellow and six green. What is the probability that a selection of five jelly beans
- contains all red jelly beans?
 - contains two red, two yellow and one green?
 - has no red beans.
14. Estimate the size of the seal population in a certain region of the Arctic if in a hunt of 50 seals 20 were tagged, and it is known that 1000 seals had been tagged before the hunt.
15. Estimate the population size of a bird species if in 100 sightings along a migration path, 25 were ringed, and it is known that 500 of the birds had been ringed before the sightings?
16. An endangered species is on the verge of extinction. Conservationist groups are closely monitoring its population each month. Last month, of 50 animals that were sighted, 20 were seen to be marked, and this month 25 of 40 animals sighted were seen to be marked. If the conservationists know that 100 of the animals in the population had been marked before the sightings, what is the estimated percentage decline in the animal population over the two months?

PROBLEMS PLUS

Consumers challenge

A school lottery is held in order to raise funds for a local charity. The rules of the lottery are as follows: 300 tickets are sold per week at \$1.00 each. Each week a winner is drawn and receives \$50.00. All the tickets for each draw are entered into the next week's draw. For example, the second week's draw contains 600 tickets, the third week, 900 tickets, and so on.

The lottery is advertised with the statement, "You get ten chances to win a one-in-three hundred draw: an overall winning chance of one in thirty!"

What is the actual probability of winning at least one \$50.00 prize if you enter the lottery on every draw?

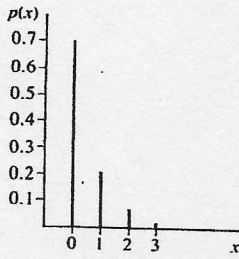
What is the expected return on your tickets?

6.6 REVIEW EXERCISE

- Identify possible random variables for the following experiments and the possible values that the random variable may take.
 - tossing a coin
 - throwing a dart
 - rolling a die
 - cutting a deck of cards
 - drawing names from a hat

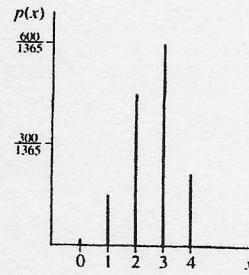
(c)

x	$p(x)$
0	0.7
1	0.21
2	0.063
3	0.0189



(iii) (a)

x	$p(x)$
0	$\frac{5}{1365}$
1	$\frac{100}{1365}$
2	$\frac{450}{1365}$
3	$\frac{600}{1365}$
4	$\frac{210}{1365}$



(b) $\frac{8}{3}$

3. $\frac{8}{81}$ 4. 0.274 5. 125 : 91 6. 364
 7. 10 8. 0.017 9. 0.556
 10. (a) 0.24 (b) 0.936
 11. (a) 32 (b) 0.141

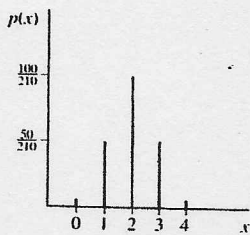
4. (a) 0.257 (b) 0.687 (c) 0.0047
 5. 0.848 6. (a) 0.253 (b) 0.019
 7. (a) 0.215 (b) 0.063
 8. 0.115 9. 0.018 10. 0.288
 11. 0.026
 12. 0.000 394
 13. (a) 0.0026 (b) 0.181 (c) 0.087
 14. 2500 15. 2000 16. 36%

EXERCISE 6.5

1. (a) hypergeometric (b) geometric
 (c) hypergeometric (d) binomial
 2. $a = 10, b = 8, n = 4, x = 2$

3. (i) (a)

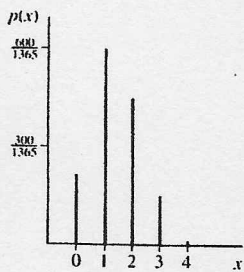
x	$p(x)$
0	$\frac{5}{210}$
1	$\frac{50}{210}$
2	$\frac{100}{210}$
3	$\frac{50}{210}$
4	$\frac{5}{210}$



(b) 2

(ii) (a)

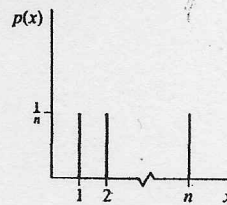
x	$p(x)$
0	$\frac{210}{1365}$
1	$\frac{600}{1365}$
2	$\frac{450}{1365}$
3	$\frac{100}{1365}$
4	$\frac{5}{1365}$



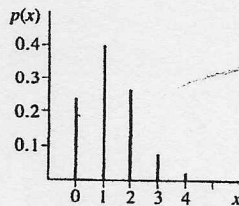
(b) $\frac{4}{3}$

6.6 REVIEW EXERCISE

3. (a)



(b)



(c)

