

Theoretical Probability

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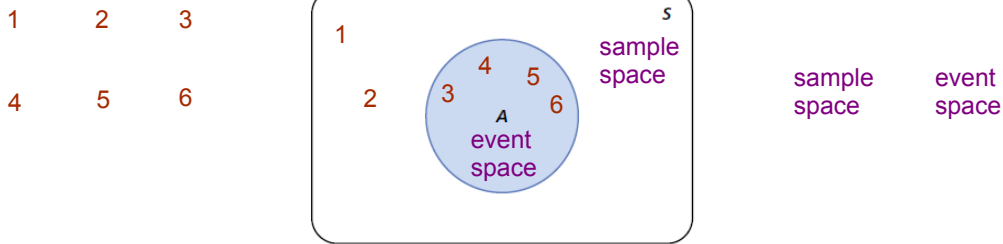
- The **theoretical probability** of an event occurs when all the outcomes of the event are equally likely. It is represented as the ratio of the number of outcomes that correspond to the event compared to the total number of outcomes.
- For example, consider the simple event (i.e., only one outcome) of rolling a number greater than 2 on a single, six-sided die.
 - > $P(\text{rolling} > 2) = \frac{4}{6}$, where 4 is the number of outcomes corresponding to the event and 6 is the total number of outcomes. In lowest form, $P(\text{rolling} > 2) = \frac{2}{3}$.

Drag this to the target to reveal the answers.

Representations for Probability

Consider the following Venn Diagram and the event of rolling > 2 :

Where would you place the following elements? Labels?



1 The number of elements corresponding to those in the **event space** is $n(A)$. The outcomes in this space are specific to the event of interest--i.e., numbers > 2 .



2 The number of elements corresponding to those in the **sample space** is $n(S)$. The outcomes in this space are specific to the total number of outcomes



3 Thus, a formula for the probability of event A occurring is given by



$$P(A) = \frac{n(A)}{n(S)}$$

The Sum of n Outcomes

- The probability of all outcomes is 1 (i.e., 100%).

- For instance, the probabilities related to the introductory example are as follows:

$$P(A) \equiv P(\text{rolling} > 2) \\ = \frac{2}{3}$$

$$P(A') \equiv P(\text{rolling} \leq 2) \\ = \frac{1}{3}$$



The **complement** of a set, A , is written as A' and consists of all the outcomes in the sample space that are not in set A .

Check Your Understanding

To do so, verify that the sum of all outcomes is 1.

$$P(A) + P(A') = \frac{2}{3} + \frac{1}{3}$$

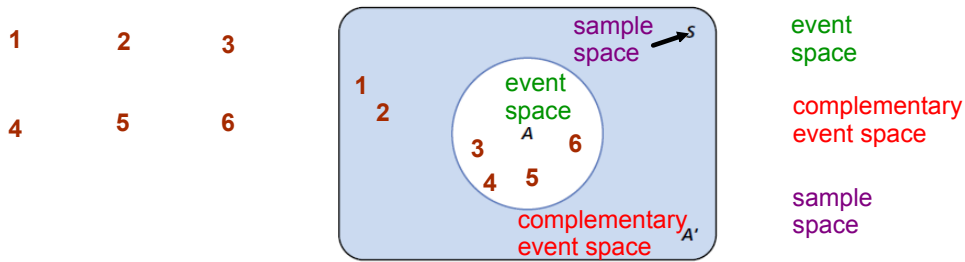
$$= \frac{3}{3}$$

$$= 1 \text{ or } 100\%$$

Complementary Events

Consider the following Venn Diagram:

Where would you place the following elements? Labels? (i.e., for the complementary event to the introductory example?)



NOTE:

- Although the Venn diagram seems to show that $n(S) = n(A')$, this is not the case. In fact, $n(A) + n(A') = n(S)$.
- Also note that the following relationship is true: $P(A') = 1 - P(A)$

Example

- Calculate the probability of rolling a sum of 7 when rolling two six-sided dice.
- Identify the complementary event to the event in a).
- Calculate the probability of not rolling a sum of 7 when rolling two six-sided dice.

a) Let A be the event of rolling a sum of 7.

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

b) $P(A') = 1 - P(A)$

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$

c) $P(\text{not rolling a sum of 7})$

$$= 1 - P(A)$$

$$= P(A')$$

$$= \frac{5}{6}$$

	D_1					
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

D_2

36 outcomes