Independent vs. Dependent Events

Lesson objectives

-to apply tree diagrams to solving problems that involve independent and dependent events

1.1

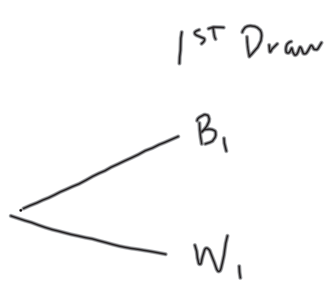
Lesson objectives | Teachers' notes | Lesson notes

Finding Probability Using Tree Diagrams

In section 4.2, you may have seen tree diagrams can be used to help compute the probabilities of combined outcomes (e.g. P(A and B)) in a sequence of experiments. The successive branches in the diagram correspond to a sequence of outcomes. Recall the homework problem where 3 coins were tossed at the same time, and you were required to determine the probability that all 3 would come up heads.

E.g.,1. Two balls are drawn in succession, without replacement, from a box containing 3 blue and 2 white balls. What is the probability of drawing a white ball on the second draw?

Solution: Start with a tree diagram showing the combined outcomes of the 2 experiments.

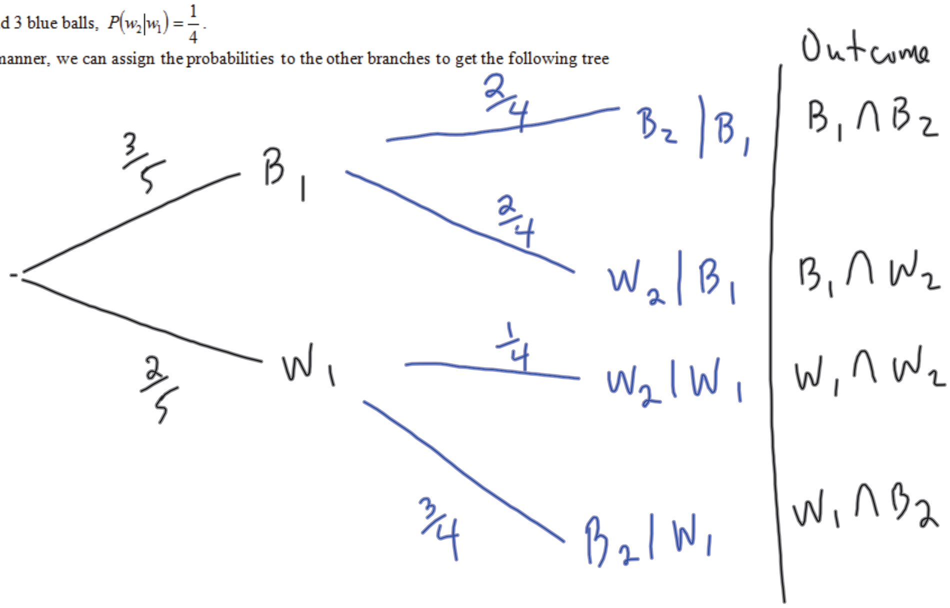


Now assign a probability to each branch on the tree.

The branch w_1 gets assigned $\frac{2}{5}$.

The branch w_1w_2 is equivalent to $P(w_2|w_1)$. This is the probability of drawing a white ball on the second draw given that a white ball was drawn on the first draw and not replaced. Since the box now contains 1 white ball and 3 blue balls, $P(w_2|w_1) = \frac{1}{4}$.

In a similar manner, we can assign the probabilities to the other branches to get the following tree diagram.



Now assign a probability to each branch on the tree.

The branch w_1 gets assigned $\frac{2}{5}$.

The branch w_1w_2 is equivalent to $P(w_2|w_1)$. This is the probability of drawing a white ball on the second draw given that a white ball was drawn on the first draw and not replaced. Since the box now contains 1 white ball and 3 blue balls, $P(w_2|w_1) = \frac{1}{4}$.

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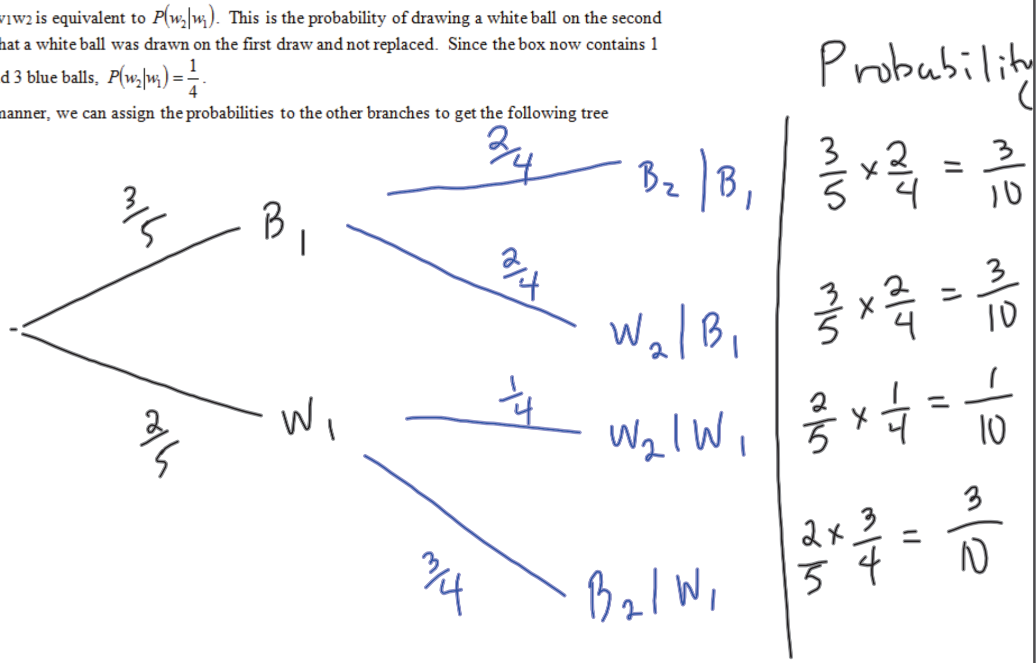


The probability of the combined outcome $w_1 \cap w_2$, by use of the product rule for conditional probability is

$$P(w_2 \cap w_1) = P(w_1) \times P(w_2 | w_1)$$

The probabilities of each of the remaining paths can be found in the same way.

Now we can complete the problem. A white ball drawn on the second draw corresponds to either the combined outcome $w_1 \cap w_2$ or $b_1 \cap w_2$ occurring. Since these combined outcomes are mutually exclusive, then

$$P(w_2) = P(w_1 \cap w_2) + P(b_1 \cap w_2)$$

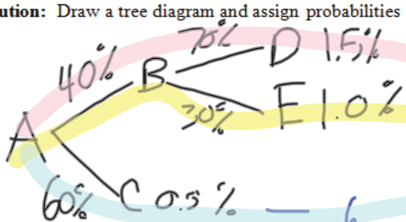
$$= \frac{1}{10} + \frac{3}{10}$$

$$= \frac{4}{10} \text{ or } \frac{2}{5}$$

which are just the probabilities listed at the ends of the two paths terminating with w_2.

E.g., 2. A large computer company A sub contracts the manufacturing of its circuit boards to two companies, 40% to company B and 60% to company C. Company B in turn subcontracts 70% of the orders it receives from company A to company D and the remaining 30% to company E, both subsidiaries of company B. When companies D, E, and C complete the boards, they are shipped to company A to be used in various computer modes. It has been found that 1.5%, 1%, and 0.5% of the boards from D, E, and C, respectively, become defective sometime during the 90-day warranty period after a computer is sold. What is the probability that a given board in a computer will be defective during the 90-day warranty period?

Solution: Draw a tree diagram and assign probabilities to each branch.



Outcomes	Probability
$A \cap B \cap D$	$\frac{21}{500}$ $\cancel{\frac{15}{100}}$ $= \frac{4}{10} \times \frac{7}{10} \times \frac{3}{20} = \frac{84}{2000} = \frac{21}{500} = 4.2\%$
$A \cap B \cap E$	$\frac{4}{10} \times \frac{3}{10} \times \frac{1}{100} = \frac{12}{10\,000} = 0.12\%$
$A \cap C$	$\frac{6}{10} \times \frac{5}{1000} = \frac{30}{10\,000} = 0.3\%$

P(Defective) =

$$= P(A \cap B \cap D) + P(A \cap B \cap E) + P(A \cap C)$$

$$= 4.2\% + 0.12\% + 0.3\%$$

$$= 4.62\%$$